PROBLEMS

10990. Proposed by Rick Mabry, LSUS, Shreveport, LA. The n+1 Bernstein polynomials of degree n are defined by

$$b_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}, \ 0 \le k \le n.$$

When all n+1 polynomials are plotted on the same graph for large fixed n over the interval $0 \le x \le 1$, an 'upper envelope' begins to be seen. The figure below shows a scaled plot of the case n = 16, with the vertical scale multiplied by 4.



Let $\beta(x) = \lim_{n \to \infty} \sqrt{n} \max_{0 \le k \le n} b_{n,k}(x)$. Find a closed form expression for $\beta(x)$.