

N3.) (8 points) Spew forth formulas. Write the standard identities for the following.

20

(a) $\cos(x + y)$

(b) $\sin(a - b)$

(c) $\tan(\alpha - \beta)$

(d) $\sin 2t$

(e) $\cos 2\theta$ (write all three!)

(f) $\sin(B/2)$

(g) $\tan(\theta/2)$ (write at least two)

(h) $\sin(\pi + x)$

N4.) (3 points) Write the *exact algebraic values* of the functions $\sin \theta$, $\cos \theta$ and $\tan \theta$ for the angle θ in standard position having the point $(-\sqrt{3}, 2)$ on its terminal side.

23

$\sin \theta =$

$\cos \theta =$

$\tan \theta =$

N5.) (6 points) Write

29

(a) a reciprocal identity involving $\cos \theta$,

(b) a Pythagorean identity involving $\cot \theta$,

(c) a cofunction identity involving $\csc \theta$.

N6.) (6 points) Write the exact values of

35

(a) $\arctan(-1)$

(b) $\arccos(-1)$

(c) $\arcsin(-1)$

N7.) (5 points) Verify the identity $\frac{\sin x - \cos x}{\cos x} + \frac{\sin x + \cos x}{\sin x} = \csc x \sec x$. Show all significant steps. 40

N8.) (8 points) Simplify each the following. Show all significant steps. 48

(a) $\frac{1 - \cos 2x}{\sin 2x}$

(b) $\tan(2 \arctan x)$

N9.) (5 points) Find an exact algebraic expression for $\cos 7\pi/12$. (Correct answers have square roots but no trig functions, no angles, no π 's.)

53

N10.) (8 points) Evaluate the following exactly.

61

(a) $\arcsin(\sin(6\pi/7))$

(b) $\arccos(\cos(6\pi/7))$

(c) $\arctan(\tan(6\pi/7))$

(d) $\arcsin(\cos(6\pi/7))$

(★) Give an approximate numerical value for $\arccos(-1/10)$ in decimal degrees. Better guesses get better scores.

The “Yes Calculators” Pages

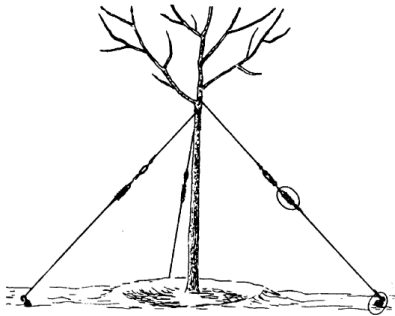
Instructions: Answer all problems correctly. Calculators are allowed on this part but *they must not be used to retrieve information or formulas.*

Put final answers in boxes when provided. Each starred problem is extra credit, and each ★ is worth 5 points.

The phrase *exact algebraic values* appears throughout the test. Quantities such as $\sqrt{3}$, $5/3$, etc., are exact algebraic values, as opposed to *numerical approximations*, such as 1.732, 1.666, etc., and *trigonometric expressions*, such as $\sin 60^\circ$ and $\tan(5\pi/8)$, which are not.

1. (5 points) A transplanted tree is made to stand up straight by running taut wires (“guy wires”) from anchors in the ground to points on the tree. Suppose that one of the wires meets the ground at an angle of 47° at a point 8.5 feet from the base of the tree. How long is the wire?

66



Guying large trees provides stability until roots grow into the soil.

From R.A. Harris, A.T. Leiser, and W.B. Davis, 1978, Staking landscape trees. Univ. Calif. Agr. Ext. Leaflet 2578.

2. (5 points) A tourist on the ground views the Eiffel tower, which has a height of 986.0 feet. The top of the tower has an angle of elevation of 24.36° . The tourist walks directly toward the tower until the distance to the base of the tower is exactly half what it was previously. What is the new angle of elevation to the top of the tower?

71

3. (6 points) A ship leaves port and travels for 5.0 miles with a bearing of $N20.0^\circ E$. The ship then turns (on a dime) and travels 5.0 more miles with a bearing of $N80.0^\circ W$. How far is the ship from port?

77

4. (5 points) The Andromeda galaxy, believed to be disk-shaped, subtends about 3 degrees field of view in the sky. (Think of six full moons touching side by side. That's huge! The galaxy is so faint that we don't see its extent without powerful telescopes, but still...) The most recent estimates of the distance to the galaxy put it about 2.9 million light-years away. What is the approximate diameter of the galaxy in light-years? 82

5. (6 points) Assuming $\cos \alpha = -2/3$ and $180^\circ < \alpha < 360^\circ$, give exact algebraic values for the following. 88

(a) $\cos(2\alpha)$

(b) $\sin(\alpha/2)$

6. (5 points) How fast is the tip of the minute hand of a clock moving (in cm/sec) if the minute hand is 18.00 cm long? 93

7. (5 points) A triangle has angles 10.0° , 20.0° and 150.0° . Its longest side (which is opposite the largest angle, of course) measures 10.00 inches. How long is its shortest side? 98

8. (5 points) A triangle has sides 99.000 cm, 100.000 cm, 101.000 cm. Find the smallest angle of the triangle to the nearest hundredth of a degree (or give it in calculator-ready form). 103

9. (6 points) Find all real solutions in $[0, 2\pi)$. Show all significant steps.

109

$$2 \cos^2 x = \cos x$$

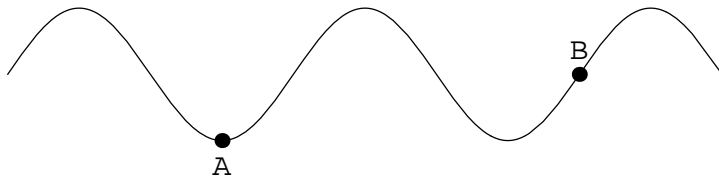
10. (6 points) Find the exact (x, y) -coordinates of the point obtained by rotating the point $(1, 2)$ counterclockwise through 60° about the origin.

115

$x =$

$y =$

11. (5 points) Find an *exact* equation that represents the sinusoidal curve pictured below. The two points are $A = (0, -1)$ and $B = (5\pi/6, 7)$. Be sure to write (separately) the values for the period and amplitude, each worth one point partial credit.



*** Extra Credit ***

(You may do these on the back of the previous page if you wish.)

- A.) (★) When the morning sun has an angle of elevation of just 20.00° , a stiff and straight stalk of beans leans directly toward the sun, making an angle of 10.00° away from the vertical. The stalk is 4.00 feet in length. How long is its shadow?

B.) (★) Use Heron's formula to find the area of the triangle in problem #8.

C.) (★★) Circles of radii 3.00, 4.00 and 5.00 inches are mutually tangent. Find the area of the gap formed between them. (I'll draw a picture on the board if you wish).

D.) (★) *Derive* the Law of Cosines formula.

E.) (★) Find the exact value of $\arcsin(\sin(7/2))$.

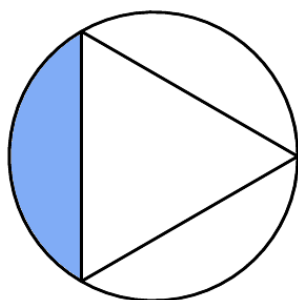
F.) (★) Find the exact value of $\arccos(\cos(30))$. (Read this very carefully.)

G.) (★) Give all exact solutions to the equation

$$6 \cos^2 x - 7 \cos x = 3$$

H.) Find the exact (x, y) -coordinates of the point obtained by rotating the point $(1, 2)$ counterclockwise through 60° about the point $(2, -1)$.

I.) ★ Consider the triangle inscribed in the circle below. If each side of the triangle has length equal to 1 unit, what is the area of the shaded region?



J.) (★...★) Ask a question (or questions!) you wish I had asked and answer it (or them). Points will certainly vary. (Don't give repeats of what I've already asked—this is where you ask me the things you were ready for but that I didn't ask!)