The "No Calculators" Pages

Instructions: See front page for general instructions. Finish this page before going to the rest. You may not return to this page once you turn on your calculator.

N1.) (12 points) Find exact algebraic values for each of the following, where defined. Otherwise, write "undefined." 12

(a) $\tan 135° = -1$

(b) $\sin 120° = \sqrt{3}/2$

(c) $\cos(-90°) = 0$

(d) $\cos(150°) = -\sqrt{3}/2$

(e) $\cot(-90°) = 0$

(f) $\tan 4\pi/3 = \sqrt{3}$

(g) $\sec 30° = 2/\sqrt{3}$

(h) $\csc 315° = -\sqrt{2}$

(i) $\cot \pi/3 = 1/\sqrt{3}$

(j) $\tan 270°$ is undefined

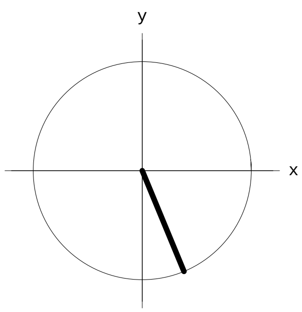
(k) $\csc 5\pi/6 = 2$

(l) $\cos 3\pi/4 = -\sqrt{2}/2$

($\star$) Give exact algebraic value for $\sin(720720360720360360720720720720300°)$.

I should give the answer to a st$\star$rred problem? I think I'll refrain! But here's a hint: $\sin(\theta + n360°) = \sin(\theta) \quad \forall n \in \mathbb{Z}$.

N2.) (8 points) An angle θ is shown is in standard position. Give approximate values for $\sin\theta$ and $\cos\theta$. 20



Just by looking, I'd say that the cosine is just less than half. The sine is nearly -1 but not quite. So I think the following are reasonable answers.

$\sin\theta \approx -.9$

$\cos\theta \approx .4$

N3.) (10 points) For each below, circle the inequality ($<$ or $>$) that makes the statement true. *Incorrect answers will be awarded negative points to discourage random guessing.* (So perhaps you should leave an answer blank if you have no idea.) 30

(a) $\sin 66° \boxed{<} > \sin 67°$

(b) $\sin 155° < \boxed{>} \sin 156°$

(c) $\sin 244° < \boxed{>} \sin 245°$

(d) $\sin 333° \boxed{<} > \sin 334°$

(e) $\cos 77° < \boxed{>} \cos 88°$

(f) $\cos 83° < \boxed{>} \cos 93°$

(g) $\tan 200° \boxed{<} > \tan 222°$

(h) $\tan 75° \boxed{<} > \tan^2 75°$

(i) $\tan 125° \boxed{<} > \sec 25°$

(j) $\sec 2° < \boxed{>} \cos 2°$

N4.) (6 points) Write the *exact algebraic values* of the functions $\sin\theta$, $\cos\theta$ and $\tan\theta$ for the angle θ in standard position having the point $(\sqrt{5}, -2)$ on its terminal side. 36

$\sin\theta = -2/3$

$\cos\theta = \sqrt{5}/3$

$\tan\theta = -2/\sqrt{5} = -2\sqrt{5}/5$

Trigonometry | Summer 2005
MATH 122-1A | Name: ____________

Test #1

Instructions: Answer all problems correctly. Calculators are allowed (except on the "No Calculators Page") but *they must not be used to retrieve information or formulas.* Freel free to leave numerical answers in "calculator-ready form." Each st⋆rred problem is extra credit, and each ⋆ is worth 5 points.

The phrase *exact algebraic values* appears throughout the test. Quantities such as $\sqrt{3}$, 5/3, etc., are exact algebraic values, as opposed to *numerical approximations*, such as 1.732, 1.666, etc., which are not.

A maximum of 115 points (out of 100) will be awarded on this test. Enjoy.

1. (6 points) The terminal side of an angle θ in standard position lies on the line $4y + 5x = 0$, with $x < 0$. Find exact algebraic values for $\cos\theta$ and $\tan\theta$. 42

$$\boxed{\cos\theta = \frac{-4}{\sqrt{41}} \qquad \tan\theta = -5/4}$$

2. (6 points) Write 48

   (a) a reciprocal identity involving $\sec\theta$, $\boxed{\sec\theta = \dfrac{1}{\cos\theta}}$

   (b) a Pythagorean identity involving $\sec\theta$, $\boxed{1 + \tan^2\theta = \sec^2\theta}$

   (c) a cofunction identity involving $\sec\theta$. $\boxed{\sec\theta = \csc(90^\circ - \theta)}$

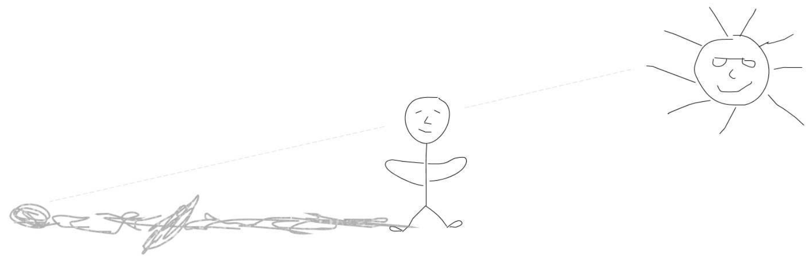
3. (9 points) Assuming $\sin\theta = 3/4$, and $\theta \in \text{QII}$, give exact algebraic values for the following. 57

   (a) $\cos\theta = -\dfrac{\sqrt{7}}{4}$

   (b) $\tan\theta = -\dfrac{3}{\sqrt{7}}$

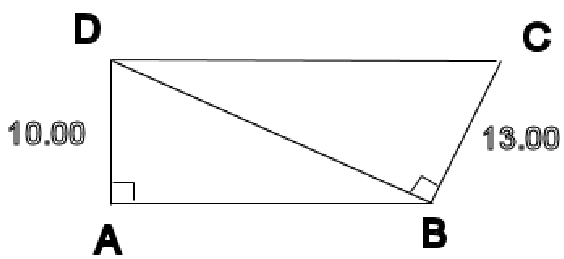
   (c) $\csc\theta = 4/3$

4. (8 points) A child standing exactly four feet tall stands in the morning sun when the sun has an angle of elevation of just 10.0°. How long is the child's shadow on the ground? 65



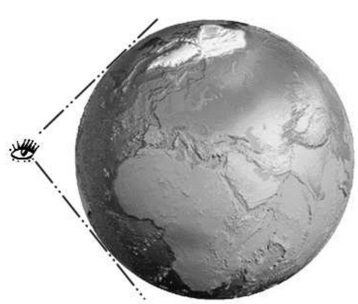
The shadow's length is $\dfrac{4}{\tan 10.0^\circ}$.

5. (8 points) In the figure below, $\angle DAB = 90^\circ$, $\angle DBC = 90^\circ$ and $\angle BCD = 70^\circ$. Find the length of segment $\overline{AB}$. 73



$AB = \sqrt{(13.00\tan 70^\circ)^2 - 10.00^2}$.

6. (8 points) At a point 1320 miles from the *surface* of the earth, how big does the earth look? (Answer this as an angle, the angular "field of view", which is the measure of the angle formed at the eye by two tangent lines drawn to opposite points on the visible disk of the sphere. You need to know the radius of the earth for this one, too. Use 4000 miles if you don't remember the more exact number, but that will cost you a point.) 81



What a cute picture! Anyway, the angular field of view α is given by

$$2\sin^{-1}\left(\frac{R}{1320+R}\right),$$

where $R = 3960$. So $\alpha = 2\sin^{-1}(3960/5280)$.

7. (8 points) A tourist views the Eiffel tower, which has a height of about 986 feet, and the tourist measures the angle of elevation top of the tower to be 13.6°. The tourist then approaches the tower directly until the distance to the base of the tower is half of what it was previously. What is the new angle of elevation to top of the tower? 89

The new angle is

$$\tan^{-1}(2\tan 13.6^\circ).$$

Incidentally, perhaps you'd guess that the result would be 2 times 13.6°. Grab a calculator and see that it isn't true.

8. (6 points) A tractor's tire measures 26.5 inches in radius. How far does the tractor move if the tire rotates through an angle of 55°? 95

$(26.5)(55\pi/180)$ inches.

9. (6 points) Two pulleys are connected by a belt. The radii of the pulleys are 8.00 inches and 12.00 inches. If the smaller pulley rotates at a rate of $\omega = 144^\circ$ per second, how fast does the larger pulley rotate? 101

It should go slower, correct? Its angular speed is $(8/12)(144^\circ$ per second$) = 96^\circ$/sec.

10. (6 points) Shreveport is at latitude 32.2°N. How far is Shreveport from the Tropic of Capricorn, which is at 23.5°S? 107

Assuming the radius of the eart is $R = 3960$ miles, the distance (as the crow or albatross flies) is

$$R(32.2 + 23.5)(\pi/180) = 55.7\ R\ \pi/180 = \frac{(55.7)(3960)\pi}{180} \text{ miles.}$$

11. (16 points) A ship leaves port and travels for 5.3 miles with a bearing of S23°E. The ship then travels 2.9 miles with a bearing of N38°E. 123

(a) How far is the ship from port?

Let $x_1 = 5.3\sin 23^\circ$, $y_1 = 5.3\cos 23^\circ$, $x_2 = 2.9\sin 38^\circ$ and $y_2 = 2.9\cos 38^\circ$, each in miles. Then the distance (in miles) is

$$\sqrt{(x_1 + x_2)^2 + (y_1 - y_2)^2}.$$

(b) What is the new bearing of the ship as measured from the port?

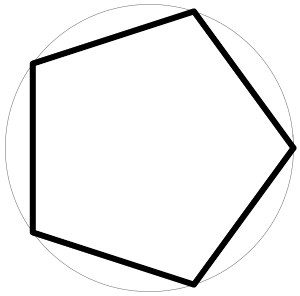
Using the same values in the answer to part (a), the bearing will be S θ E, where

$$\theta = \tan^{-1}\left(\frac{x_1 + x_2}{y_1 - y_2}\right).$$

⋆⋆⋆ Extra Credit ⋆⋆⋆

(You may do these on the back of the previous page if you wish.)

A.) (⋆) Find the area A of a regular pentagon inscribed in a circle of radius R.



Hint: Draw center. Make triangles. Find angles. Find areas. Sum areas.

B.) (⋆) In problem #5, suppose that instead of specifying $\angle BCD$, we are told only that segments $\overline{CD}$ and $\overline{AB}$ are parallel. Find the length of $\overline{AB}$.

It works out to $AB = \mathbf{100/\sqrt{69}}$.

C.) (⋆) Find any solution (x, y) to the following equation.

$$\cos(4x + y) = \csc(2y + 22.0^\circ)$$

Cute problem. I don't recall that anyone has ever solved it on a test. Hint: there are very few ways a cosine can equal a cosecant.

D.) (⋆) A parallelogram has sides measuring 33.3 inches and 22.2 inches, and has an acute angle of 11.1°. Find the length of the short diagonal of the parallelogram.

It's $\sqrt{(22.2 \sin 11.1^\circ)^2 + (33.3 - 22.2 \cos 11.1^\circ)^2}$ inches.

E.) (⋆) Write an equation for the line passing through the point $(2, 3)$ and making a *counter-clockwise* angle of 43° with respect to the positive y-axis (with the positive y-axis as the initial side of the angle).

$$y = 3 + (\tan 47^\circ)(x - 2)$$

F.) ($\star$) Prove the following classical fact of geometry: On a circle, let A and B denote the endpoints of a diameter. Let C denote any other point on the circle. Then the angle at C in $\triangle ABC$ is a right angle. (You can use some trig to prove this but there is a way to do it by simply summing angles in triangles.)

(Draw the picture. But don't simply mark the picture and expect me to assemble the pieces or guess your thoughts. You must speak some mathematics. For instance, ...) Let O denote the center of the circle. Consider the segment OC and observe that $\triangle AOC$ and $\triangle COB$ are isosceles (the equal sides being the radii of the circle). Let $O_1 = \angle AOC$ and $O_2 = \angle COB$. Then since the angles in any triangle sum to $180°$, we have $2A + O_1 = 180°$ and $2B + O_2 = 180°$. Adding these equations, we have

$$2A + 2B + O_1 + O_2 = 360°.$$

But since $O_1 + O_2 = 180°$, we have

$$2A + 2B = 180°,$$

hence $A + B = 90°$. Since $A + B = C$, we're done. ▲

G.) ($\star \cdots \star$) Ask a question you wish I had asked and answer it. Points will vary.

I don't give many points for easy questions or "repeats" (i.e., questions that are minor variations of a question I have actually asked on the test), but I cannot understand why more people don't have something ready for this question, since it is on every test I give. I can't ask everything — this is a chance to show me you know something different from what I have already asked on the test, perhaps something you studied that you want to show off.