Trigonometry
MATH 122-1A

Summer 2005
Name:

Test #2 (Answers)

Instructions: Answer all problems correctly. Calculators are allowed but *they must not be used to retrieve information or formulas.* Each st⋆rred problem is extra credit, and each ⋆ is worth 5 points.
A maximum of 115 points (out of 100) will be awarded on this test.

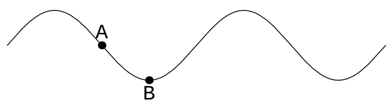
1. (12 points) Consider the graph of the function $y = -1 + 2\sin(3x - \pi/4)$. 12

   (a) What is the exact value of the amplitude of this function? $\boxed{A = 2}$

   (b) What is the exact value of the period? $\boxed{P = 2\pi/3}$

   (c) Give the exact (x, y) coordinates for some maximum of the curve. $\boxed{(x, y) = (\pi/4, 1)}$

2. (12 points) Consider the function whose graph, a sinusoidal curve, is below. The coordinates of the points shown are $A = (\pi/3, 2)$ and $B = (\pi/2, -1)$. 24



   (a) What is the exact value of the amplitude of this function? $\boxed{3}$

   (b) What is the exact value of the period? $\boxed{2\pi/3}$

   (c) Write an exact formula for the function. $\boxed{y = 2 - 3\sin(3(x - \pi/3))}$
   Another correct answer is $y = 2 - 3\cos(3(x - \pi/2))$. Better still: $y = 2 + 3\sin 3x$.

3. (16 points) Spew forth some formulas. (Write the standard identities for the following.) 40

   (a) $\sin(x - y)$

   (b) $\cos(a - b)$

   (c) $\tan(\alpha - \beta)$

   (d) $\sin 2\theta$

   (e) $\cos 2A$ (write all three identities)

   (f) $\sin(x/2)$

   (g) $\cos(\theta/2)$

   (h) $\tan(A/2)$ (write at least two)

   You'll look these up.

4. (12 points) Simplify the following $\boxed{52}$

(a) $\sin(-x)$ $\boxed{= -\sin x}$

(b) $\tan(\pi + B)$ $\boxed{= \tan B}$

(c) $\sin(90^\circ + y)$ $\boxed{= \cos y}$

(d) $\cos(\pi - x)$ $\boxed{= -\cos x}$

(e) $\cos(270^\circ - \theta)$ $\boxed{= -\sin\theta}$

(f) $\tan(\pi/2 + A)$ $\boxed{= -\cot A}$

5. (12 points) Assuming $\cos\theta = -\dfrac{2}{3}$ and $\sin\beta = \dfrac{1}{\sqrt{2}}$ and that $\theta \in$ QII and $\beta \in$ QII, give exact algebraic values for the following. $\boxed{64}$

(a) $\cos(\theta - \beta)$ $\boxed{= \dfrac{2\sqrt{2} + \sqrt{10}}{6}}$

(b) $\tan(\theta - \beta)$ $\boxed{= \dfrac{2 - \sqrt{5}}{2 + \sqrt{5}} = 4\sqrt{5} - 9}$

6. (12 points) Assuming $\cos\alpha = -2/3$ and $180^\circ < \alpha < 360^\circ$, give exact algebraic values for the following. $\boxed{76}$

(a) $\cos(2\alpha)$ $\boxed{=1/9}$

(b) $\cos(\alpha/2)$ $\boxed{= -1/\sqrt{6}}$

7. (12 points) Assuming $\sin\alpha = -3/5$ and $180^\circ < \alpha < 360^\circ$, give exact algebraic values for the following. $\boxed{88}$

(a) $\sin(2\alpha)$ $\boxed{= \pm 24/25 \text{ (both are possible)}}$

(b) $\sin(\alpha/2)$ $\boxed{= \dfrac{1}{\sqrt{10}} \text{ or } \dfrac{3}{\sqrt{10}} \text{ (both are possible)}}$

8. (6 points) Write the number $\boxed{94}$

$$\cos 170^\circ \sin 60^\circ - \sin 170^\circ \cos 60^\circ$$

in the form of a single trig function of a single exact angle.

$\boxed{\sin(-110^\circ)}$

9. (6 points) Find an exact algebraic expression for $\cos 75^\circ$. (Use a sum-formula with some familiar angles or a half-angle formula.) $\boxed{100}$

$\boxed{\dfrac{\sqrt{6} - \sqrt{2}}{4}}$

10. (6 points) Find an exact algebraic expression for $\tan 5\pi/8$. (Use a half-angle formula.) 106

$\boxed{-1-\sqrt{2}}$

11. (6 points) Verify. 112

$$\frac{\tan A - \cot A}{\sec A + \csc A} = \sin A - \cos A$$

$$\begin{aligned} \text{LHS} &= \frac{\dfrac{\sin A}{\cos A} - \dfrac{\cos A}{\sin A}}{\dfrac{1}{\cos A} + \dfrac{1}{\sin A}} = \frac{\dfrac{\sin^2 A - \cos^2 A}{\cos A \sin A}}{\dfrac{\sin A + \cos A}{\cos A \sin A}} = \frac{(\sin A - \cos A)(\sin A + \cos A)}{\sin A + \cos A} \\ &= \text{RHS} \end{aligned}$$

12. (6 points) Verify. 118

$$\frac{2\tan x}{1+\tan^2 x} = \sin 2x$$

$$\begin{aligned} \text{LHS} &= \frac{2\tan x}{1+\tan^2 x} \cdot \frac{\cos^2 x}{\cos^2 x} \\ &= \frac{2\sin x \cos x}{\cos^2 x + \sin^2 x} = \frac{\sin 2x}{1} = \text{RHS} \end{aligned}$$

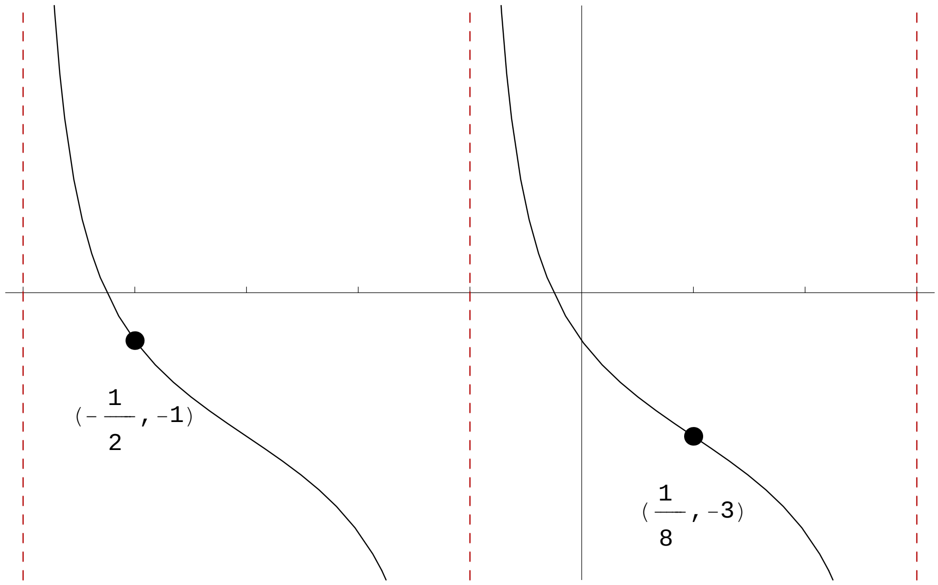
13. (6 points) Simplify. 124

$$\frac{1-\cos 2x}{\sin 2x} = \cdots = \tan x$$

⋆⋆⋆ Extra Credit ⋆⋆⋆

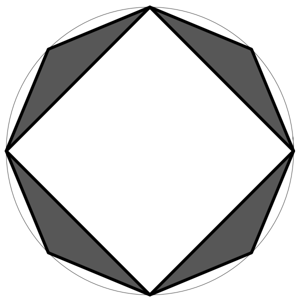
(You may do these on the back of the previous page if you wish.)

A.) (⋆) Find an equation that matches the graph.



$y = -3 + \cot(2\pi(x - 1/8))$

B.) (⋆) The shaded region has area equal to 1 square unit. What is the radius of the circle?



$r = \dfrac{1}{\sqrt{2\sqrt{2} - 2}}$ (Forms of the answer will vary greatly.)

C.) (⋆) The line $y = 2x$ is rotated counterclockwise about the origin through an angle of 30°. Find the exact algebraic value of the slope of the line obtained.

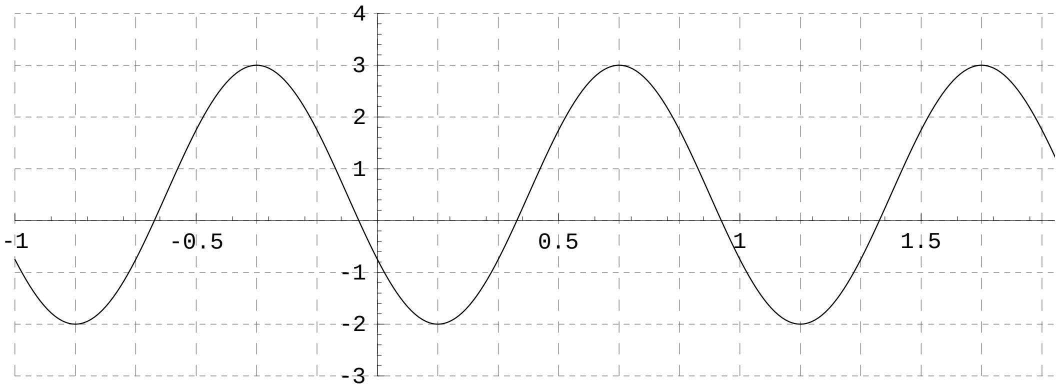
$m = -8 - 5\sqrt{3}$

D.) (⋆) Prove the following classical fact of geometry: On a circle, let A and B denote the endpoints of a diameter. Let C denote any other point on the circle. Then the angle at C in $\triangle ABC$ is a right angle. (You can use some trig to prove this but there is a way to do it by simply summing angles in triangles.)

E.) ($\star$) Prove the following classical fact of geometry: Let A, B and C denote any points on a circle centered at the point O. Prove that $m(\angle ACB) = \dfrac{1}{2}\, m(\angle AOB)$.

Do this "triglessly" — use an angle-countling proof.

F.) ($\star$) Consider the function whose graph, a sinusoidal curve, is below.



(a) What is the exact value of the amplitude of this function? $\boxed{A = 5/2}$

(b) What is the exact value of the period? $\boxed{P = 1}$

(c) Write an exact formula for the function. $\boxed{y = \dfrac{1}{2} + \dfrac{5}{2}\cos\left(2\pi\left(x + \dfrac{1}{3}\right)\right)}$

G.) ($\star$) Write at least two of the "product-to-sum" identities.

Look 'em up.

H.) ($\star \cdots \star$) Ask a question you wish I had asked and answer it. Points will vary.