Trigonometry   Summer 2005

MATH 122-1A   Name:

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Test #2

Instructions: Answer all problems correctly. Calculators are allowed but *they must not be used to retrieve information or formulas.* Each st⋆rred problem is extra credit, and each ⋆ is worth 5 points. A maximum of 115 points (out of 100) will be awarded on this test.

1. (12 points) Consider the graph of the function $y = -1 + 2\sin(3x - \pi/4)$.   `12`
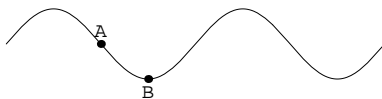
   (a) What is the exact value of the amplitude of this function?

   (b) What is the exact value of the period?

   (c) Give the exact (x, y) coordinates for some maximum of the curve.

2. (12 points) Consider the function whose graph, a sinusoidal curve, is below. The coordinates   `24`
   of the points shown are $A = (\pi/3, 2)$ and $B = (\pi/2, -1)$.

   (a) What is the exact value of the amplitude of this function?

   (b) What is the exact value of the period?

   (c) Write an exact formula for the function.

3. (16 points) Spew forth some formulas. (Write the standard identities for the following.) $\boxed{40}$

(a) $\sin(x - y)$

(b) $\cos(a - b)$

(c) $\tan(\alpha - \beta)$

(d) $\sin 2\theta$

(e) $\cos 2A$ (write all three identities)

(f) $\sin(x/2)$

(g) $\cos(\theta/2)$

(h) $\tan(A/2)$ (write at least two)

4. (12 points) Simplify the following

(a) $\sin(-x)$

(b) $\tan(\pi + B)$

(c) $\sin(90° + y)$

(d) $\cos(\pi - x)$

(e) $\cos(270° - \theta)$

(f) $\tan(\pi/2 + A)$

5. (12 points) Assuming $\cos\theta = -\dfrac{2}{3}$ and $\sin\beta = \dfrac{1}{\sqrt{2}}$ and that $\theta \in$ QII and $\beta \in$ QII, give exact algebraic values for the following.

(a) $\cos(\theta - \beta)$

(b) $\tan(\theta - \beta)$

6. (12 points) Assuming $\cos\alpha = -2/3$ and $180° < \alpha < 360°$, give exact algebraic values for the following.

    (a) $\cos(2\alpha)$

    (b) $\cos(\alpha/2)$

7. (12 points) Assuming $\sin\alpha = -3/5$ and $180° < \alpha < \mathbf{270°}$, give exact algebraic values for the following.

    (a) $\sin(2\alpha)$

    (b) $\sin(\alpha/2)$

8. (6 points) Write the number

$$\cos 170° \sin 60° - \sin 170° \cos 60°$$

in the form of a single trig function of a single exact angle.

9. (6 points) Find an exact algebraic expression for $\cos 75°$. (Use a sum-formula with some familiar angles or a half-angle formula.)

10. (6 points) Find an exact algebraic expression for $\tan 5\pi/8$. (Use a half-angle formula.)

11. (6 points) Verify.

$$\frac{\tan A - \cot A}{\sec A + \csc A} = \sin A - \cos A$$

12. (6 points) Verify.

$$\frac{2\tan x}{1+\tan^2 x} = \sin 2x$$

13. (6 points) Simplify.

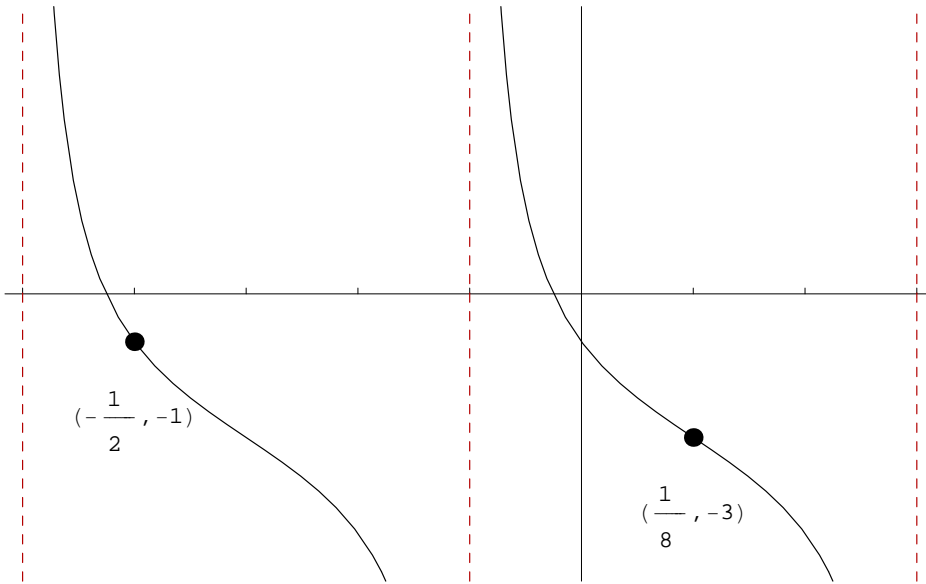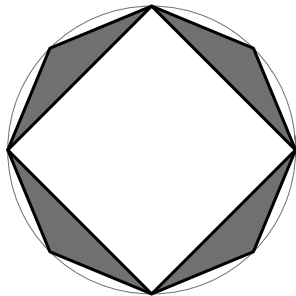$$\frac{1-\cos 2x}{\sin 2x}$$

(You may do these on the back of the previous page if you wish.)

A.) (★) Find an equation that matches the graph.

$$(-\frac{1}{2}, -1)$$

$$(\frac{1}{8}, -3)$$

B.) (★) The shaded region has area equal to 1 square unit. What is the radius of the circle?

C.) (★) The line $y = 2x$ is rotated counterclockwise about the origin through an angle of $30°$. Find the exact algebraic value of the slope of the line obtained.

D.) (★) Prove the following classical fact of geometry: On a circle, let A and B denote the endpoints of a diameter. Let C denote any other point on the circle. Then the angle at C in $\triangle ABC$ is a right angle. (You can use some trig to prove this but there is a way to do it by simply summing angles in triangles.)

E.) (★) Prove the following classical fact of geometry: Let A, B and C denote any points on a circle centered at the point O. Prove that $m(\angle ACB) = \dfrac{1}{2} m(\angle AOB)$.

F.) (⋆) Consider the function whose graph, a sinusoidal curve, is below.
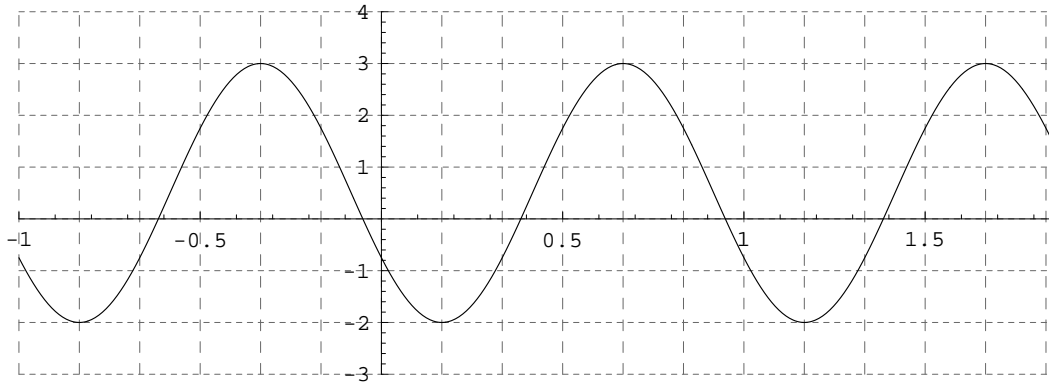
(a) What is the exact value of the amplitude of this function?

(b) What is the exact value of the period?

(c) Write an exact formula for the function.

G.) (⋆) Write at least two of the "product-to-sum" identities.

H.) (⋆⋯⋆) Ask a question you wish I had asked and answer it. Points will vary.