

Test #3

Instructions: Answer all problems correctly. Calculators are NOT allowed. Each starred problem is extra credit, and each ★ is worth 5 points. A maximum of 115 points (out of 100) will be awarded on this test.

1. (16 points) Spew forth some formulas. (Write the standard identities for the following.)

16

(a) $\sin(x - y)$

(b) $\cos(a - b)$

(c) $\tan(\alpha - \beta)$

(d) $\sin 2\theta$

(e) $\cos 2A$ (write all three identities)

(f) $\sin(x/2)$

(g) $\cos(\theta/2)$

(h) $\tan(A/2)$ (write at least two)

2. (6 points) (Thes again...) Simplify the following

22

(a) $\sin(-x)$

(b) $\tan(\pi + B)$

(c) $\sin(90^\circ + y)$

(d) $\cos(\pi - x)$

(e) $\cos(270^\circ - \theta)$

(f) $\tan(\pi/2 + A)$

3. (12 points) Assuming $\cos \theta = \frac{3}{4}$ and $\sin \beta = \frac{1}{\sqrt{2}}$ and that $\theta \in \text{QI}$ and $\beta \in \text{QII}$, give exact algebraic values for the following.

34

(a) $\cos(\theta + \beta)$

(b) $\tan(\theta - \beta)$

4. (12 points) Assuming $\cos \alpha = -2/3$ and $180^\circ < \alpha < 360^\circ$, give exact algebraic values for the following. 46

(a) $\cos(2\alpha)$

(b) $\cos(\alpha/2)$

5. (12 points) Assuming $\sin \alpha = -3/5$ and $180^\circ < \alpha < 270^\circ$, give exact algebraic values for the following. 58

(a) $\sin(2\alpha)$

(b) $\sin(\alpha/2)$

6. (8 points) Write the number

66

$$\cos 170^\circ \sin 60^\circ - \sin 170^\circ \cos 60^\circ$$

in the form of a single trig function of a single exact angle.

7. (8 points) Find an exact algebraic expression for $\cos 75^\circ$. (Use a sum-formula with some familiar angles or a half-angle formula.)

74

8. (8 points) Find an exact algebraic expression for $\tan 5\pi/8$. (Use a half-angle formula.)

82

9. (8 points) Verify.

90

$$\frac{\tan A - \cot A}{\sec A + \csc A} = \sin A - \cos A$$

10. (8 points) Verify.

98

$$\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$$

11. (8 points) Simplify.

$$\frac{1 - \cos 2x}{\sin 2x}$$

106

12. (8 points) Find the exact algebraic coordinates of the point obtained by rotating the point $(2, -1)$ about the origin, counterclockwise through 60° .

114

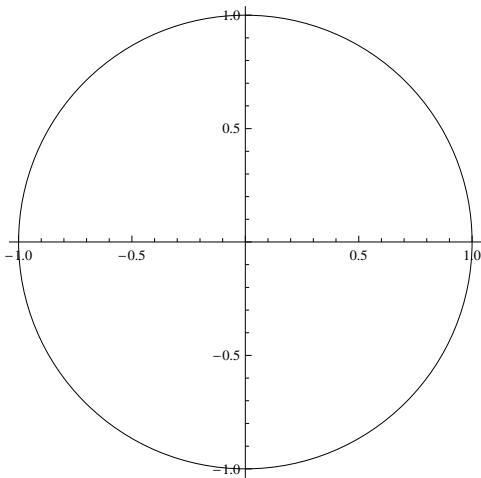
*** Extra Credit ***

(You may do these on the back of the previous page if you wish.)

A.) (★) Derive the product-to-sum formula for $\cos x \cos y$.

B.) (★) The line $y = 2x$ is rotated counterclockwise about the origin through an angle of 60° . Use identities involving tangents of sums to find the exact algebraic value of the slope of the line obtained.

C.) (★) Approximate, as accurately as you can, the degree measure of $\sin^{-1}(3/5)$. Two points are lost for each degree off. A brief explanation of your reasoning could increase your score.



D.) (★...★) Ask a question you wish I had asked and answer it. Points will vary.