Calc I Fall 2008
MATH 221-001 Name:

T2

Instructions: Do all problems correctly. You may NOT use calculators or any electronic devices or notes of any kind. Loads of points are possible on the test, but the highest grade that I will award is 110 points.

1. (8 points) Data dump! Purge for credit! Write the indicated formulas. 8

   (a) the product rule for derivatives

   (b) the quotient rule for derivatives

   (c) the chain rule for derivatives

   (d) the two equivalent limits that yield the derivative $f'(a)$

2. (8 points) More dumping! Write the formulas for each derivative. 16

   (a) $\frac{d}{dx}\cos x$

   (b) $\frac{d}{dx}x^n$

   (c) $\frac{d}{dx}\log x$

   (d) $\frac{d}{dx}\tan x$

   (e) $\frac{d}{dx}\arctan x$

   (f) $\frac{d}{dx}e^x$

   (g) $\frac{d}{dx}a^x$ (when a is a constant)

   (h) $\frac{d}{dx}\sec x$

3. (8 points) Find $f'(x)$ if $f(x) = \dfrac{x^2}{3x^4 - 2x + 1}$. 24

4. (8 points) Find y' if $y = \sqrt{3x - 1}\,\ln(x^2 + x)$. 32

5. (8 points) Find $\dfrac{d}{dx}\,(3x - \cos(5x))^4$. 40

6. (8 points) Find $g'(t)$ when $g(t) = \dfrac{e^{-x^2}}{\pi^2 + 7}$. 48

7. (8 points) Use a (limit) definition of the derivative to evaluate $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right)$. 56

8. (8 points) Use the quotient rule (or power rule), the trig identity for $\sec x$ in terms of $\cos x$ (you'd better know it), and the formula you gave in problem #2a (let's hope it is correct) to derive the formula for the derivative of $\sec x$. (If you aren't sure what I'm asking for here, my advice is to skip this one.) 64

9. (8 points) Evaluate $\dfrac{d}{dx} \ln \left(\dfrac{(2x-1)^7}{(3x^2+1)^5} \right)$. 72

10. (8 points) Evaluate $\dfrac{d}{dx}\left(x^2+1\right)^{1/x}$. 80

11. (8 points) Use implicit differentiation to find y' (in terms of x and y) if 88

$$xy^2 - e^{3y} = 7x.$$

12. (8 points) Find an equation for the slope of the line tangent to the curve 96

$$x^3 + y^3 = 3x - y + 4$$

at the point $(2, 1)$.

13. (6 points) Evaluate $\lim_{t \to 0} \dfrac{\sin^2(3t)}{t \sin(5t)}$. 102

14. (8 points) Derive the formula for the derivative of $\arccos x$. 110

15. (8 points) State the theorem relating continuity to differentiability. (This was nicknamed the "Joe Biden Theorem" in class, for reasons I cannot recall. You may *prove* the theorem for extra credit — see problem E.) 118

⋆ ⋆ ⋆ Extras ⋆ ⋆ ⋆

Each st⋆rred problem is extra credit and each ⋆ is worth 5 points.

(Feel free to do these on the back of the previous page or elsewhere. Just tell me where to look.)

A. (⋆) Find a formula, in terms of x and the positive integer n, for the n'th derivative $f^{(n)}(x)$ of the function $f(x) = x\,e^x$. For a few extra points, use the function $f(x) = x\,e^{ax}$ instead, where a is a constant.

B. (⋆⋆) Consider the curve $x^3 + y^3 = 3x - y + 4$ given in problem #12.

(a) Show that the curve has two horizontal tangents.

(b) Show that the curve has no vertical tangents.

(c) Find the exact (x, y) coordinates for *one* of the horizontal tangents.

C. (⋆) One-sided derivatives.

(a) Write the formal (limit) definition of $f'_+(x)$, the right-hand derivative of a function $f(x)$.

(b) The function $f(x) = |x^2 - x| + 2x$ fails to be differentiable at $x = 0$ and at $x = 1$, but the one-sided derivatives exist there. Calculate (using any method you wish) the left-hand and right-hand derivatives $f'_-(x)$ and $f'_+(x)$ at $x = 0$.

D. (⋆) The *symmetric derivative* of a function $f(x)$ is defined to be

$$f'_s(x) = \lim_{h\to 0} \frac{f(x+h) - f(x-h)}{2h},$$

provided the limit exists. Calculate $f'_s(0)$ for the function $f(x) = |x^2 - x| + 2x$.

E. (⋆) Prove the Joe Biden Theorem. (See problem #15.)

F. (⋆) Prove that a differentiability implies symmetric differentiability. That is, prove that if $f'(a)$ exists, then $f'_s(a)$ also exists.

G. (⋆) Give an example of a function that is differentiable but whose derivative is not continuous.

H. (⋆) Evaluate the limit by using the trick of converting to appropriate derivatives (perhaps after inserting a convenient form of the number 1). Show all work. No credit if you use theorems from later in the course.

$$\lim_{h\to 0} \frac{2^h - 1}{\cos 2h - 1}$$

I. (⋆ $\cdots$ ⋆) Surely I forgot something you were ready for. Ask a question you wish I had asked and answer it. Points may vary. Offer void where prohibited by law.