Calc I Fall 2008
MATH 221-001 Name:

T3

Instructions: Do all problems correctly. You may NOT use calculators or any electronic devices or notes of any kind. Loads of points are possible on the test, but the highest grade that I will award is 115 points. Each ⋆ is extra credit and worth 5 points.

1. (20 points) Do one of the following. (You may do the other for one ⋆ extra credit.) You may leave your answers in "calculator-ready" form. (That is, you needn't simplify your answers, but they must be numeric. However, a simplified answers is worth 3 extra points.) **20**

   (a) At noon, ship A is 30 km due west of ship B. Ship A then sails due south while ship B sails due north. A short time later, ship A has traveled 15 km and ship B has sailed 25 km, and at that moment, ship A is moving at 35 km/hr, and ship B is traveling 5 km/hr. How fast is the distance between the two ships changing at that instant? [Note: the speeds of the ships are not constant.]

   (b) A light on an ambulance rotates 3 times per second (about a vertical axis, like a lighthouse.). The ambulance's light throws a spot (of light) onto a long, straight wall that runs due north and south. The point P on the wall that is nearest the light is 20 meters from the light. How fast along the wall (in meters per second) is the spot moving along the wall when it is 10 meters from P?

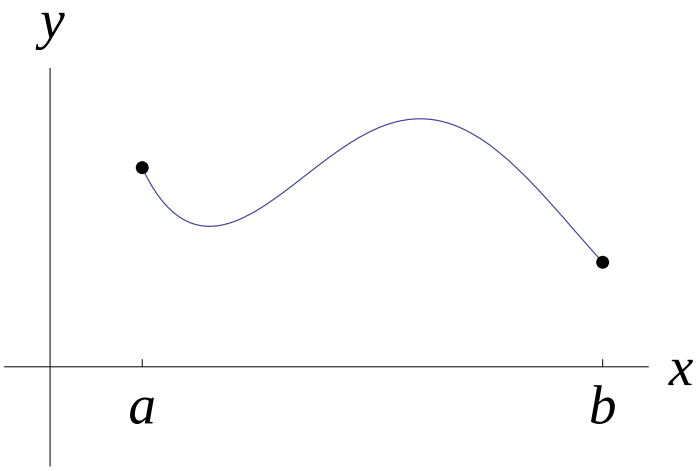
2. (8 points) For $y = 1/(x^2 + 1)$, find the differential dy and evaluate dy with the values $x = 3$ and $dx = 1/10$ (and simplify). 28

($\star$) Use differentials to give a formula for the approximate relative error in the calculation of the volume of a sphere when the radius is r and the error in measuring r is Δr.

3. (12 points) Find the absolute maximum and minimum values of the function $q(x) = x^4 - 2x^2 + 3$ on the interval $[-2, 3]$. 40

4. (5 points) State the Mean Value Theorem (including all hypotheses and conclusions). 45

5. (6 points) Mark the graph below at the point(s) $(c, f(c))$ where c satisfies the conclusions of the Mean Value Thorem on the interval $[a, b]$. (Of course you're approximating this visually. Get as close as you can, so I'll know you understand.) 51



6. (10 points) For the function $f(x) = x^3 + x$ on the interval $[-2, 0]$, find all values of c (if any) satisfied by the conclusion of the Mean Value Theorem. 61

7. (6 points) Give the definition of a *local maximum* of a function. **67**

8. (6 points) State the theorem called the *First Derivative Test*. **73**

9. (6 points) State the theorem called the *Increasing/Decreasing Test*. (It should be called the *Monotonicity Test*, if you ask me.) **79**

10. (6 points) Give the definition of *concave upward*. **85**

11. (6 points) State the theorem called the *Concavity Test*. **91**

12. (20 points) Choose (and circle) one of the two functions below for this problem. 111

(i) $g(x) = 3x^{2/3} - 2x$

(ii) $h(x) = (x-2)^3(x+1)$

For the selected function, do the following.

(a) Find the intervals of increase or decrease.

(b) Find the local max and min values (if any).

(c) Find the intervals of concavity and inflection points (if any).

(d) Sketch the graph. (Make your own x- and y-axes, but be neat.)

⋆ ⋆ ⋆ Extras ⋆ ⋆ ⋆

Each st⋆rred problem is extra credit and each ⋆ is worth 5 points.

(Feel free to do these on the back of the previous page or elsewhere. Just tell me where to look.)

A. (⋆) Evaluate $\dfrac{d}{dx}\cosh(x^2 + x)$.

B. (⋆) A population of *Calcucoccus disorientus* bacteria are growing exponentially. The population triples every 7 hours. Given an intial population of one million *C. disorientus*, write a formula for the number at time t (in hours). When are there two million of these vile cooties?

C. (⋆) Find the absolute maximum and minimum values of the function $L(x) = x - \ln x$ on the interval $\left[\frac{1}{2}, \frac{3}{2}\right]$. (The trick is to do this without a calculator. You must explain your answers — not much credit otherwise. The *better* the explanation, the more credit given. A hint, if you can make sense of it: there is some symmetry in the numbers you must "test" but no symmetry in the derivative. I've said too much...)

D. (⋆ $\cdots$ ⋆) Surely I forgot something you were ready for. Ask a question you wish I had asked and answer it. Points may vary. Offer void where prohibited by law.