Calc II
MATH 222

Fall 2005
Name:

Final Exam

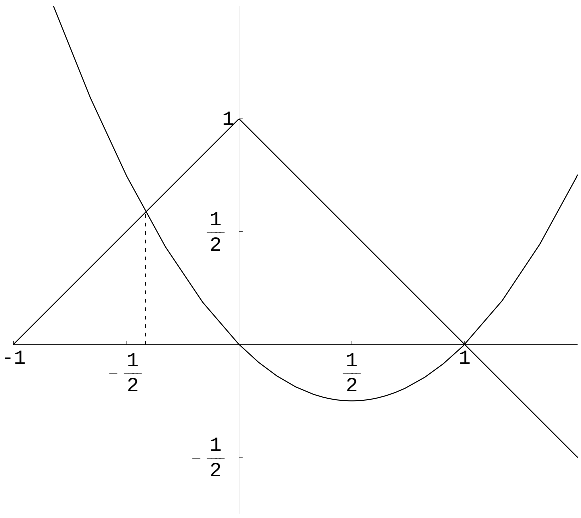
Instructions: Write answers to problems *on separate paper.* You may NOT use calculators or any electronic devices or notes of any kind. Loads of points are possible on the test, but the highest grade that I will award is 120 points.

Unless otherwise specified, you may leave **definite** integrals unevaluated ("just set it up"), but they must be "ready to evaluate," that is, each must be a definite integral involving one variable only (e.g., no mixed x's and y's) with **explicit functions and explicit limits of integration and no absolute values whatsoever**.

1. (6 points) Find the area bounded by $y = 1 - |x|$ and $y = x^2 - x$. (Set it up.)

   Here's the graph (along with the *Mathematica* code that produced it, in case you're interested):

```
Plot[{1 - Abs[x], x^2 - x}, {x, -2, 2},
 PlotRange → {{-1, 1.5}, {-.75, 1.5}}, Epilog →
  {Dashing[{.005, .01}], Line[{{1 - Sqrt[2], 0}, {1 - Sqrt[2], 2 - Sqrt[2]}}]},
 AspectRatio → Automatic, Ticks → {{-1, -1/2, 1/2, 1}, {-1/2, 1/2, 1}}]
```



   Write $y = 1 - |x|$ as

$$y = \begin{cases} y = 1 - x & \text{if} \quad x \geq 0 \\ y = 1 + x & \text{if} \quad x < 0 \end{cases}$$

   and solve the resulting equations. Setting $1 - x = x^2 - x$ you'll get $x = \pm 1$, of which (as can be seen), only $x = 1$ applies; setting $1 + x = x^2 - x$, you solve a quadratic to get $x = 1 \pm \sqrt{2}$ and you'll use the negative root only (noted in the graph as a dashed line; $1 - \sqrt{2} \approx -0.414214$). The area bounded by the curves therefore lies between $x = 1 - \sqrt{2}$ and $x = 1$, with the parabola below the absolute value function. So the area is

$$\begin{aligned} A &= \int_{1-\sqrt{2}}^{1} ((1-|x|) - (x^2 - x))\, dx \\ &= \int_{1-\sqrt{2}}^{0} (1+2x - x^2)\, dx + \int_{0}^{1} (1-x^2)\, dx, \end{aligned}$$

and that's how you may leave it. (The result is actually $-1 + 4\sqrt{2}/3$.)

2. (6 points) Use the method of disks/washers to find the volumes of the solids obtained by rotating the region bounded by the curves $y = 2x^2$ and $y = x^3$ about the following line $y = -2$. Set it up (but it's not hard to evaluate).

   The curves intersect where $x = 0, 2$ and it should be clear that the parabola is above the cubic in the interval between these values. (There are various ways to see this, but here's how I know it without writing anything down: *cubic beats quadratic*. How must I be thinking to utter such a cryptic statement?) A washer between x and $x + dx$ has thickness, or height dh equal to dx and has inner radius $r = x^3 + 2$ and the outer radius $R = 2x^2 + 2$. So the volume is

$$\begin{aligned} V &= \int_a^b \pi(R^2 - r^2) dh \\ &= \int_0^2 \pi \left((2x^2+2)^2 - (x^3+2)^2\right)\, dx. \end{aligned}$$

3. (6 points) Same as above but use shells.

   For shells we need a thickness dr, a radius r and a height h. Here, $0 \le y \le 8$ and for each y we have $dr = dy$, $r = y + 2$ and $h = \sqrt[3]{y} - \sqrt{y/2}$. We have then

$$\begin{aligned} V &= \int_a^b 2\pi r h dr \\ &= \int_0^8 2\pi(y+2)\left(\sqrt[3]{y} - \sqrt{y/2}\right)\, dy. \end{aligned}$$

4. (6 points) Find the average value of the function $y = x^2$ on the interval $[a, b]$. (Evaluate completely to get an exact algebraic answer.)

   By the definition,

$$\begin{aligned} \bar{f} &= \frac{1}{b-a} \int_a^b f(x)\, dx \\ &= \frac{1}{b-a} \int_a^b x^2\, dx \\ &= \frac{1}{b-a} \frac{b^3 - a^3}{3} \\ &= \frac{b^2 + ab + a^2}{3} \end{aligned}$$

5. (4 points) State the precise hypotheses and conclusion of the Mean Value Theorem for Integrals.
   Look it up.

6. (6/5/5/4/4 points) Evaluate each of the following integrals.

   (a) $\int x\cos(5x)\,dx$
   By parts, with $u = x$ and $dv = \cos(5x)dx$.

$$\begin{aligned}\int x\cos(5x)\,dx &= x\left(\frac{\sin 5x}{5}\right) - \int \frac{\sin 5x}{5}\,dx \\ &= x\left(\frac{\sin 5x}{5}\right) + \frac{\cos 5x}{25} + C.\end{aligned}$$

   (b) $\int \ln(7x)\,dx$
   By parts again, with $u = \ln(7x)$ and $dv = dx$. Note that $du = \dfrac{7}{7x}\,dx = \dfrac{1}{x}\,dx$.

$$\begin{aligned}\int \ln(7x)\,dx &= x\ln(7x) - \int (x \cdot \frac{1}{x})\,dx \\ &= x\ln(7x) - x + C.\end{aligned}$$

Another way to handle it is to simply recall that $\int \ln x\,dx = x\ \ln x - x + C$, and that using our "linear argument trick" we get

$$\frac{1}{7}(7x\ \ln(7x) - 7x) + C.$$

Or, beating this to death, we also have that $\ln(7x) = \ln 7 + \ln x$, so

$$\begin{aligned}\int \ln(7x)\,dx &= \int (\ln 7 + \ln x)\,dx \\ &= (\ln 7)x + x\ \ln x - x + C,\end{aligned}$$

and you should quickly verify this is equivalent to the earlier version.

(c) $\int \sin^4(5x)\, dx$

Basic trig identities give

$$\begin{aligned}
\sin^4\theta &= \left(\frac{1-\cos(2\theta)}{2}\right)^2 \\
&= \frac{1}{4}\left(1 - 2\cos(2\theta) + \cos^2(2\theta)\right) \\
&= \frac{1}{4} - \frac{1}{2}\cos(2\theta) + \frac{1}{4}\left(\frac{1+\cos(4\theta)}{2}\right) \\
&= \frac{3}{8} - \frac{1}{2}\cos(2\theta) + \frac{1}{8}\cos(4\theta).
\end{aligned}$$

This has an easy antiderivative,

$$\frac{3}{8}\theta - \frac{1}{4}\sin(2\theta) + \frac{1}{32}\sin(4\theta) + C,$$

so our integral becomes

$$\begin{aligned}
\int \sin^4(5x)\, dx &= \frac{1}{5}\left(\frac{3}{8}5x - \frac{1}{2}\sin(10x) + \frac{1}{32}\sin(20x)\right) + C \\
&= \frac{3x}{8} - \frac{1}{20}\sin(10x) + \frac{1}{160}\sin(20) + C
\end{aligned}$$

(d) $\int \sqrt{4+9x^2}\, dx$.

Trig substitution works well. Get $9x^2 = 4\tan^2\theta$ by letting $x = \frac{2}{3}\tan\theta$. Then $dx = \frac{2}{3}\sec^2\theta\, d\theta$ and we have

$$\begin{aligned}
\int \sqrt{4+9x^2}\, dx &= \int \sqrt{4+4\tan^2\theta}\,\frac{2}{3}\sec^2\theta\, d\theta \\
&= \int \frac{4}{3}\sec^3\theta\, d\theta \\
&= \frac{4}{3}\cdot\frac{1}{2}(\sec\theta\ \tan\theta + \ln(\sec\theta + \tan\theta)) + C \\
&= \frac{2}{3}\left(\frac{1}{2}\sqrt{4+9x^2}\,\frac{3x}{2} + \ln(\frac{1}{2}\sqrt{4+9x^2} + \frac{3x}{2})\right) + C \\
&= \frac{1}{2}x\,\sqrt{4+9x^2} + \frac{2}{3}\ln(3x + \sqrt{4+9x^2}) + C,
\end{aligned}$$

where we have used the fact that $\sec\theta = \frac{1}{2}\sqrt{4+9x^2}$ and we've alse "re-used" the arbitrary constant C. Of course, we have also ignored the issue of absolute values in the above. (You're welcome.)

(e) $\displaystyle\int \frac{2x^3 - x}{x^2 - 4}\, dx$

Partial fractions are easy here (trig substitution could also be used). First divide and then use partial fractions to get

$$\frac{2x^3 - x}{x^2 - 4} = 2x + \frac{7x}{x^2 - 4} = 2x + \frac{7/2}{x-2} + \frac{7/2}{x+2}.$$

The integral is then

$$x^2 + \frac{7}{2}(\ln(x-2) + \ln(x+2)) + C = x^2 + \frac{7}{2}\ \ln(x^2 - 4) + C.$$

7. (3 points each) Evaluate each definite integral completely, if possible, and simplify. Otherwise, if the integral diverges, say so and state how it diverges (e.g., to ∞ or $-\infty$ or otherwise) *and explain your conclusion.*

(a) $\displaystyle\int_{-1}^{1} \frac{dx}{x^4}$

The integrand is discontinuous at $x = 0$ so we use improper integrals:

$$\begin{aligned} \int_{-1}^{1} \frac{dx}{x^4} &= \int_{-1}^{0} \frac{dx}{x^4} + \int_{0}^{1} \frac{dx}{x^4} \\ &= \lim_{T\to 0-} \int_{-1}^{T} \frac{dx}{x^4} + \lim_{T\to 0+} \int_{T}^{1} \frac{dx}{x^4} \end{aligned}$$

Each of the above limits is easily seen to diverge to $+\infty$, so the entire integral does, as well.

(b) $\displaystyle\int_{-\infty}^{0} \frac{x\, dx}{2x^2 + 1}$

It is easy to see that

$$\begin{aligned} \int_{-\infty}^{0} \frac{x\, dx}{2x^2 + 1} &= \lim_{T\to -\infty} \left(\frac{1}{4} \ln(2x^2 + 1) \Big|_{T}^{0} \right) \\ &= 0 - \infty \\ &= -\infty \end{aligned}$$

8. (6 points) A thin chain hangs in the shape of a catenary given by $y = \cosh 3x$, between $x = 0$ and $x = 1$. Find the length of the chain. Recall that $\cosh t = \frac{1}{2}(e^t + e^{-t})$ and $\sinh t = \frac{1}{2}(e^t - e^{-t})$. (Set it up.)

   Perhaps you notice or recall that $\frac{d}{dt}\cosh t = \sinh t$. Regardless, you should get $\frac{dy}{dx} = 3\sinh 3x = \frac{3}{2}(e^{3x} - e^{-3x})$, so just plugging these into the formula for arclength, we get

$$\begin{aligned} L &= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \\ &= \int_0^1 \sqrt{1 + (3\sinh(3x))^2}\, dx = \int_0^1 \sqrt{1 + \left(\frac{3}{2}(e^{3x} - e^{-3x})\right)^2}\, dx \end{aligned}$$

   A note and apology: an actual catenary would not be of this form; I need another constant. The curve $y = \frac{1}{3}\cosh(3x)$ is a catenary. That constant would actually let you compute the definite integral above, as opposed to merely setting it up. Check it out.

9. (4 points) Whip that chain! Rotate the chain in problem #8 about the line $x = 3$ and find the area of the resulting surface.

   The area of $y = f(x)$ for $x \in [a, b]$ rotated around a vertical axis $x = c$ is $A = \int_a^b 2\pi r ds$, where r is the distance from a point on the curve to the axis of rotation, and $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$. So,

$$A = \int_0^1 (3 - x)\sqrt{1 + (3\sinh(3x))^2}\, dx.$$

10. (6 points) Find the centroid of the region bounded by the graph of the curves $y = x^2$ and $x = y^2$. (Evaluate the integral(s) and find the actual point.)

   By symmetry, we'll have $\bar{x} = \bar{y}$. The area of the region is

$$A = \int_0^1 (\sqrt{x} - x^2)dx = \frac{2}{3} - \frac{1}{3} = 1/3.$$

   The moment in the direction of x is

$$M = \int_0^1 x(\sqrt{x} - x^2)dx = \frac{2}{5} - \frac{1}{4} = 3/20.$$

   So

$$\bar{x} = \frac{M}{A} = \frac{3/20}{1/3} = \frac{9}{20}.$$

   The centroid is the point $\left(\frac{9}{20}, \frac{9}{20}\right)$.

11. (1 point each) Determine whether each of the following **sequences** $\{a_n\}$ converges or diverges. If it converges, find its limit. If it diverges to $+\infty$, say so. If it diverges to $-\infty$, say so. If it diverges in some other way, say how. No credit for "diverges" or "converges", but no penalties for incorrect answers.

(a) $\left\{\dfrac{2}{\ln(n+1)}\right\}$ $\quad \to \mathbf{0}$

(b) $\left\{\dfrac{n!}{(2n+1)!}\right\}$ $\quad \to \mathbf{0}$

(c) $\left\{\dfrac{n^{3n}}{3^n}\right\}$ $\quad \to \infty$

(d) $\left\{\arctan\left(\dfrac{n}{n+1}\right)\right\}$ $\quad \to \boldsymbol{\pi/4}$

(e) $\left\{\dfrac{2n^2}{(n+1)\sqrt{n^2+1}}\right\}$ $\quad \to \mathbf{2}$

(f) $\left\{\dfrac{(-2)^{2n}}{5^n}\right\}$ $\quad \to \mathbf{0}$

(g) $\left\{\cos\dfrac{(-1)^n}{n}\right\}$ $\quad \to \mathbf{1}$

(h) $\left\{\dfrac{n^3}{3^n}\right\}$ $\quad \to \mathbf{0}$

(i) $\left\{\dfrac{n(n+2)}{n+1}\right\}$ $\quad \to \infty$

(j) $\left\{\dfrac{n^{3n}}{3^{n^2}}\right\}$ $\quad \to \mathbf{0}$

For (j), notice that $\dfrac{n^{3n}}{3^{n^2}} = \left(\dfrac{n^3}{3^n}\right)^n$ and consult (h).

12. (3 points) State the precise hypotheses and conclusion of the theorem we call "the Alternating Series Test".

Look it up.

13. (2 points for each correct answer, -1 for each incorrect answer, no penalty for blanks) Determine whether each of the following series is **abs**olutely convergent, **cond**itionally convergent or **div**ergent.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2^n+e}{e^n+2}$ $\quad$ **abs**

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n^2}{2^n}$ $\quad$ **abs**

(c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n^e}{e^n}$ $\quad$ **abs**

(d) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2n}{3n-1}$ $\quad$ **div**

(e) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2^{3n}}{3^{2n}}$ $\quad$ **abs**

(f) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n!}{(n+1)!}$ $\quad$ **cond**

(g) $\displaystyle\sum_{n=1}^{\infty}\tan\frac{(-1)^n}{n}$ $\quad$ **cond**

(h) $\displaystyle\sum_{n=1}^{\infty}\arcsin\frac{3(-1)^n n+1}{2n^3+n}$ $\quad$ **abs**

(i) $\displaystyle\sum_{n=1}^{\infty}\cos\frac{(-1)^n(n+1)}{2n^3+3}$ $\quad$ **div**

(j) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(\ln n)^3}{n^2}$ $\quad$ **abs**

(k) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\ln n}{n^2}$ $\quad$ **abs**

(l) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(\ln n)^3}{n}$ $\quad$ **cond**

(m) $\displaystyle\sum_{n=1}^{\infty}\frac{(-2)^n}{n!}$ $\quad$ **abs**

(n) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n^n}{n!}$ $\quad$ **div**

(o) $\displaystyle\sum_{n=1}^{\infty}\sinh\left(\frac{(-1)^n}{n^2+1}\right)$ $\quad$ **abs**

14. (3 points each) Find the intervals of convergence of each of the power series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x)^n}{n^2}$ $\quad [\mathbf{-1/2, 1/2}]$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$ $\quad [\mathbf{0, 2})$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(2x)^n}{(2n)!}$ $\quad (-\infty, \infty)$

(d) $\displaystyle\sum_{n=0}^{\infty} (2n)!(x-3)^n$ $\quad \{\mathbf{3}\}$

15. (4 points each) Write down (you needn't derive it if you can just write it) the power series expansion (about $x = 0$) for each of the functions below. State the radius of convergence of each.

(a) e^{3x}

$$= 1 + 3x + \frac{3^2x^2}{2!} + \frac{3^3x^3}{3!} + \cdots \quad ; r = \infty$$

(b) $\displaystyle\frac{1}{2-3x}$

$$= \frac{1}{2(1-3x/2)} = (1/2)\left(1 + (3x/2) + (3x/2)^2 + (3x/2)^3 + \cdots\right) \quad ; r = 2/3$$

(c) $\arctan 7x$

$$\begin{aligned} \arctan t &= \int_0^t \frac{du}{1+u^2} \\ &= \int_0^t \left(1 - u^2 + u^4 - u^6 + - + - \cdots\right) du \\ &= t - \frac{t^3}{3} + \frac{t^5}{5} - \frac{t^7}{7} + - + - \cdots, \end{aligned}$$

with radius of convergence 1. So our series is just

$$7x - \frac{7^3x^3}{3} + \frac{7^5x^5}{5} - \frac{7^7x^7}{7} + - + - \cdots,$$

with $r = 1/\sqrt{7}$.

16. (3 points each) Give the first three nonzero terms of the Taylor series for the following. Again, you needn't derive it if you can just write it.

(a) x^2, centered at $c = 2$

Perhaps you noted that a Taylor series centered at c for an n'th degree polynomial is an n'th degree polynomial in $x - c$. Even if you didn't, that's what you got. You can take the derivatives and get it just as easily, but for fun, I'll do it by inspection.

$$\begin{aligned} x^2 &= (x-2)^2 + 4x - 4 \\ &= (x-2)^2 + 4(x-2) + 4 \end{aligned}$$

(b) $\sin 5x$, centered at $c = \pi/2$

Hmm, again, I could take some easy derivatives, but instead, I'll do something shifty — by noticing that

$$\begin{aligned} \sin(5(x+\pi/2)) &= \sin(5x + 5\pi/2) \\ &= \cos 5x \\ &= 1 - \frac{5^2}{2!}x^2 + \frac{5^4}{4!}x^4 - \frac{5^6}{6!}x^6 + - + - \cdots, \end{aligned}$$

we have

$$\begin{aligned} \sin 5x &= \sin(5((x-\pi/2)+\pi/2)) \\ &= 1 - \frac{5^2}{2!}(x-\pi/2)^2 + \frac{5^4}{4!}(x-\pi/2)^4 - \frac{5^6}{6!}(x-\pi/2)^6 + - + - \cdots, \end{aligned}$$

(c) $e^x \cos x$, centered at $c = 0$

Take derivatives (yuck!) or multiply series (yuck!). We just need to collect a few terms, so it can be done without all the folowing writing. The x^2 term disappears, so to get three terms we need to go as high as x^3. This can actually be done by inspecting the first and last lines below.

$$\begin{aligned} e^x \cos x &= (1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots)(1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + - + - \cdots) \\ &= (1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + - + - \cdots) \\ &\quad + x(1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + - + - \cdots) \\ &\quad + \frac{1}{2!}x^2(1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + - + - \cdots) \\ &\quad + \frac{1}{3!}x^3(1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + - + - \cdots) \\ &\quad + \cdots \\ &= (1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + - + - \cdots) \\ &\quad + (x - \frac{1}{2!}x^3 + \frac{1}{4!}x^5 - \frac{1}{6!}x^7 + - + - \cdots) \\ &\quad + (\frac{1}{2!}x^2 - \frac{1}{2!}\frac{1}{2!}x^4 + \frac{1}{2!}\frac{1}{4!}x^6 - \frac{1}{2!}\frac{1}{6!}x^8 + - + - \cdots) \\ &\quad + (\frac{1}{3!}x^3 - \frac{1}{3!}\frac{1}{2!}x^5 + \frac{1}{3!}\frac{1}{4!}x^7 - \frac{1}{3!}\frac{1}{6!}x^9 + - + - \cdots) \\ &\quad + \cdots \\ &= 1 + x + \left(\frac{1}{3!} - \frac{1}{2!}\right)x^3 + \cdots \\ &= 1 + x - \frac{1}{3}x^3 + \cdots \end{aligned}$$

Or you could use the formula for multiplication of series:

$$\left(\sum_{n=0}^{\infty} a_n x^n\right)\left(\sum_{n=0}^{\infty} b_n x^n\right) = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} a_k b_{n-k}\right)x^n.$$

(d) $\sqrt[3]{1+x}$, centered at $c=0$

This is the binomial theorem.

$$\begin{aligned}\sqrt[3]{1+x} &= 1+\frac{1}{3}x+\frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2!}x^2+\frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{-5}{3}\right)}{3!}x^3+\cdots \\ &= 1+\frac{1}{3}x-\frac{1}{9}x^2+\cdots\end{aligned}$$

17. (4 points each) Write parameterizations for each of the curves described below. Be sure to give precise intervals for the parameter (e.g., $t \in [0,2]$ or $0 \le \theta < 2\pi$, etc.).

(a) The line segment joining the points $(-2,3)$ and $(1,5)$.

A segment joining (x_0,y_0) and (x_1,y_1) can be parameterized as

$$x(t)=x_0+t(x_1-x_0), \quad y(t)=y_0+t(y_1-y_0); \quad 0 \le t \le 1.$$

So here we can write

$$x(t)=-2+3t, \quad y(t)=3+2t; \quad 0 \le t \le 1.$$

(b) The bottom half of the semicircle of radius 4 and centered at $(-2,3)$

a circle of radius r centered at (h,k) can be parameterized by

$$x(t)=h+r\cos t, \quad y(t)=k+r\sin t; \quad 0 \le t < 2\pi,$$

which starts at the extreme right of the circle (when $t=0$) and goes counterclockwise one time around. So an easy way to parameterize the bottom half of our circle is

$$x(t)=-2+4\cos t, \quad y(t)=3+4\sin t; \quad \pi \le t \le 2\pi.$$

18. (6 points) Consider the polar curve $r = 4+3\sin\theta$. At what exact values of t is there a horizontal tangent?

Since a horizontal implies $y'(t) = 0$ (for a differentiable curve) and since $y(t) = r(t)\sin t$, we have

$$\begin{aligned} y'(t) &= r'(t)\sin t + r(t)\cos t \\ &= (3\cos t \sin t + (4+3\sin t)\cos t \\ &= \cos t(4+6\sin t), \end{aligned}$$

which is zero when either $\cos t = 0$ or $\sin t = -2/3$, hence

$$t \in \{\pi/2,\ 3\pi/2,\ -\arcsin(2/3),\ \pi + \arcsin(2/3)\},$$

or any values of t obtained from these by adding multiples of 2π. Have a look.

```
{x[t_], y[t_]} = (4 + 3 Sin[t]) {Cos[t], Sin[t]};
ParametricPlot [{x[t], y[t]}, {t, 0, 2 Pi},
 AspectRatio → Automatic, Epilog → {PointSize[.02],
   Point[{x[#], y[#]}] & /@ {Pi / 2, 3 Pi / 2, -ArcSin[2 / 3], Pi + ArcSin[2 / 3]}}]
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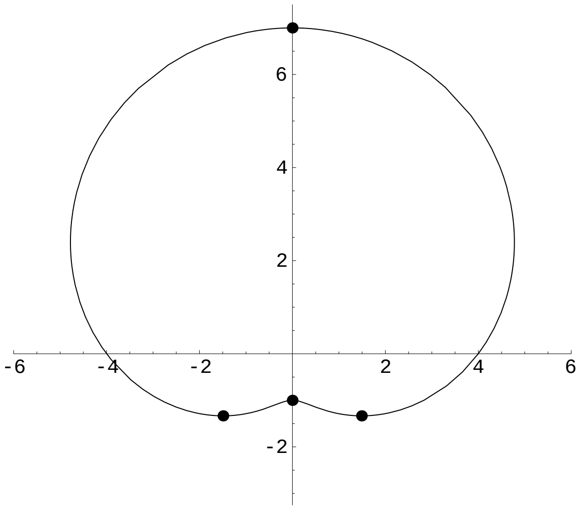
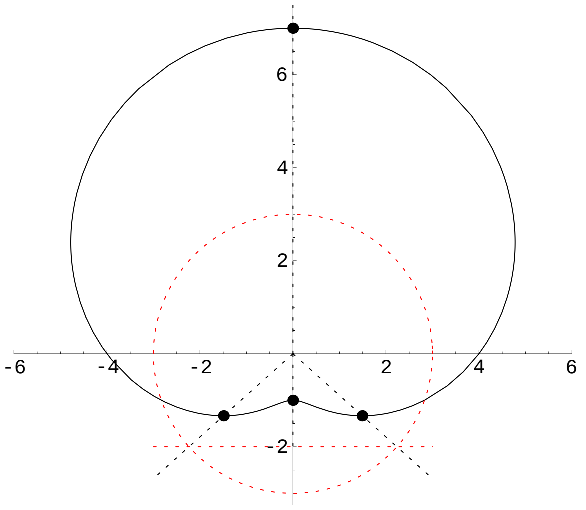


The following graphic, showing some dashed radial lines (black) along with a circle of radius three and a line at $y = -2$ (both in red), is just for fun, to show the angles t for which $\sin t = -2/3$ do what they are alleged to.



19. (6 points) What are (a) the area and (b) the perimeter of the figure in problem #18? (Set up the integrals.)

The easy area formula for a polar curve $r = f(\theta)$ is just $A = \int \frac{1}{2} r^2(\theta)\, d\theta$, and we have

$$A = \int_0^{2\pi} \frac{1}{2}(4 + 3\sin\theta)^2 \, d\theta.$$

The formula for arclength is $L = \int \sqrt{(r(\theta))^2 + (r'(\theta))^2}\, d\theta$, giving

$$L = \int_0^{2\pi} \sqrt{(4 + 3\sin\theta)^2 + (3\cos\theta)^2}\, d\theta.$$

Sorry, no extras this time! Well, just one:

($\star \cdots \star$) Ask a question you wish I had asked and answer it. Points may vary. Offer void where prohibited by time!