Calc II | Fall 2005

MATH 222 | Name: ____________

Final Exam

Instructions: Write answers to problems *on separate paper.* You may NOT use calculators or any electronic devices or notes of any kind. Loads of points are possible on the test, but the highest grade that I will award is 120 points.

Unless otherwise specified, you may leave **definite** integrals unevaluated ("just set it up"), but they must be "ready to evaluate," that is, each must be a definite integral involving one variable only (e.g., no mixed x's and y's) with **explicit functions and explicit limits of integration and no absolute values whatsoever**.

1. (6 points) Find the area bounded by $y = 1 - |x|$ and $y = x^2 - x$. (Set it up.)
2. (6 points) Use the method of disks/washers to find the volumes of the solids obtained by rotating the region bounded by the curves $y = 2x^2$ and $y = x^3$ about the following line $y = -2$. Set it up (but it's not hard to evaluate).
3. (6 points) Same as above but use shells.
4. (6 points) Find the average value of the function $y = x^2$ on the interval $[a, b]$. (Evaluate completely to get an exact algebraic answer.)
5. (4 points) State the precise hypotheses and conclusion of the Mean Value Theorem for Integrals.
6. (6/5/5/4/4 points) Evaluate each of the following integrals.
   (a) $\int x \cos(5x)\, dx$
   (b) $\int \ln(7x)\, dx$
   (c) $\int \sin^4(5x)\, dx$
   (d) $\int \sqrt{4 + 9x^2}\, dx$.
   (e) $\displaystyle\int \frac{2x^3 - x}{x^2 - 4}\, dx$

7. (3 points each) Evaluate each definite integral completely, if possible, and simplify. Otherwise, if the integral diverges, say so and state how it diverges (e.g., to ∞ or $-\infty$ or otherwise) *and explain your conclusion.*

   (a) $\displaystyle\int_{-1}^{1} \frac{dx}{x^4}$

   (b) $\displaystyle\int_{-\infty}^{0} \frac{x\,dx}{2x^2+1}$

8. (6 points) A thin chain hangs in the shape of a catenary given by $y = \cosh 3x$, between $x = 0$ and $x = 1$. Find the length of the chain. Recall that $\cosh t = \frac{1}{2}(e^t + e^{-t})$ and $\sinh t = \frac{1}{2}(e^t - e^{-t})$. (Set it up.)

9. (4 points) Whip that chain! Rotate the chain in problem #8 about the line $x = 3$ and find the area of the resulting surface.

10. (6 points) Find the centroid of the region bounded by the graph of the curves $y = x^2$ and $x = y^2$. (Evaluate the integral(s) and find the actual point.)

11. (1 point each) Determine whether each of the following **sequences** $\{a_n\}$ converges or diverges. If it converges, find its limit. If it diverges to $+\infty$, say so. If it diverges to $-\infty$, say so. If it diverges in some other way, say how. No credit for "diverges" or "converges", but no penalties for incorrect answers.

   (a) $\left\{\dfrac{2}{\ln(n+1)}\right\}$

   (b) $\left\{\dfrac{n!}{(2n+1)!}\right\}$

   (c) $\left\{\dfrac{n^{3n}}{3^n}\right\}$

   (d) $\left\{\arctan\left(\dfrac{n}{n+1}\right)\right\}$

   (e) $\left\{\dfrac{2n^2}{(n+1)\sqrt{n^2+1}}\right\}$

   (f) $\left\{\dfrac{(-2)^{2n}}{5^n}\right\}$

   (g) $\left\{\cos\dfrac{(-1)^n}{n}\right\}$

   (h) $\left\{\dfrac{n^3}{3^n}\right\}$

   (i) $\left\{\dfrac{n(n+2)}{n+1}\right\}$

   (j) $\left\{\dfrac{n^{3n}}{3^{n^2}}\right\}$

12. (3 points) State the precise hypotheses and conclusion of the theorem we call "the Alternating Series Test".

13. (2 points for each correct answer, -1 for each incorrect answer, no penalty for blanks) Determine whether each of the following series is **abs**olutely convergent, **cond**itionally convergent or **div**ergent.

(a) $\sum_{n=1}^{\infty}(-1)^n \frac{2^n+e}{e^n+2}$

(b) $\sum_{n=1}^{\infty}(-1)^n \frac{n^2}{2^n}$

(c) $\sum_{n=1}^{\infty}(-1)^n \frac{n^e}{e^n}$

(d) $\sum_{n=1}^{\infty}(-1)^n \frac{2n}{3n-1}$

(e) $\sum_{n=1}^{\infty}(-1)^n \frac{2^{3n}}{3^{2n}}$

(f) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{(n+1)!}$

(g) $\sum_{n=1}^{\infty} \tan \frac{(-1)^n}{n}$

(h) $\sum_{n=1}^{\infty} \arcsin \frac{3(-1)^n n+1}{2n^3+n}$

(i) $\sum_{n=1}^{\infty} \cos \frac{(-1)^n(n+1)}{2n^3+3}$

(j) $\sum_{n=1}^{\infty} \frac{(-1)^n(\ln n)^3}{n^2}$

(k) $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n^2}$

(l) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(\ln n)^3}{n}$

(m) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

(n) $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!}$

(o) $\sum_{n=1}^{\infty} \sinh\left(\frac{(-1)^n}{n^2+1}\right)$

14. (3 points each) Find the intervals of convergence of each of the power series.

(a) $\sum_{n=1}^{\infty} \frac{(2x)^n}{n^2}$

(b) $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$

(c) $\sum_{n=0}^{\infty} \frac{(2x)^n}{(2n)!}$

(d) $\sum_{n=0}^{\infty} (2n)!(x-3)^n$

15. (4 points each) Write down (you needn't derive it if you can just write it) the power series expansion (about $x=0$) for each of the functions below. State the radius of convergence of each.

(a) e^{3x}

(b) $\frac{1}{2-3x}$

(c) $\arctan 7x$

16. (3 points each) Give the first three nonzero terms of the Taylor series for the following. Again, you needn't derive it if you can just write it.

    (a) x^2, centered at $c = 2$

    (b) $\sin 5x$, centered at $c = \pi/2$

    (c) $e^x \cos x$, centered at $c = 0$

    (d) $\sqrt[3]{1+x}$, centered at $c = 0$

17. (4 points each) Write parameterizations for each of the curves described below. Be sure to give precise intervals for the parameter (e.g., $t \in [0, 2]$ or $0 \le \theta < 2\pi$, etc.).

    (a) The line segment joining the points $(-2, 3)$ and $(1, 5)$.

    (b) The bottom half of the semicircle of radius 4 and centered at $(-2, 3)$

18. (6 points) Consider the polar curve $r = 4 + 3\sin\theta$. At what exact values of t is there a horizontal tangent?

19. (6 points) What are (a) the area and (b) the perimeter of the figure in problem #18? (Set up the integrals.)

Sorry, no extras this time! Well, just one:

($\star \cdots \star$) Ask a question you wish I had asked and answer it. Points may vary. Offer void where prohibited by time!