Calc II
MATH 222

Fall 2005
Name: ______________

# T1

**Instructions:** Write answers to problems *on separate paper.* You may NOT use calculators or any electronic devices or notes of any kind. Each st$\star$rred problem is extra credit and each $\star$ is worth 5 points. (These are just more problems, but harder. They're worth fewer points so that you're not unduly tempted.) Loads of points are possible on the test, but the highest grade that I will award is 115 points.

Unless otherwise specified, you may leave **definite** integrals unevaluated ("just set it up"), but they must be "ready to evaluate," that is, each must be a definite integral involving one variable only (e.g., no mixed $x$'s and $y$'s) with **explicit functions and explicit limits of integration and no absolute values whatsoever**. However, if you would like to evaluate the integrals, they are worth an extra 4 points each. (*Warning:* Save these for last; some may be too difficult.)

**Diminishing returns**: Phrases such as "8/6/4 points" (see problem #3, for instance) refer to the points awarded for doing several parts of a problem. The example here indicates that 8 points will be awarded if any one problem of the three is done correctly, $8 + 6$ points if any two are correct, $8 + 6 + 4$ points for all three.

1. Find the exact areas of the regions described below. (Do not merely write integrals in this one; evaluate completely to find the exact numerical result.)
   (a) (5 points) the region bounded by the curves $y = x^2 - x$ and $y = x + 1$
   (b) (10 points) the region bounded by the curves $y = x^3 - x^2$ and $y = x^2 + x$
2. (10 points) Find the area bounded by $y = |x - 1|$ and $y = x^2 - 2$. (Set it up. Warning: you'll need the quadratic formula to find the limits. But you won't be evaluating the integral(s) so that part is no big deal.)
3. (10/6/3 points) Use the method of disks/washers to find the volumes of the solids obtained by rotating the region bounded by the curves $y = x^2$ and $y = 2x$ about each of the following lines. (Set up the integrals.)
   (a) the $x$-axis
   (b) the $y$-axis
   (c) the line $y = -1$

4. (10/4 points) Use the method of shells to find the volume of the solid obtained by rotating the region bounded by $y = x^2 - 2x + 3$ and $y = x + 3$ about each of the following lines. (Set up the integrals.)

   (a) the $y$-axis

   (b) the line $x = -2$

5. (10 points) A solid's base is a semicircle $\mathcal{S}$ of radius $R$. (If $R$ scares you, let $R = 2$, but it will cost you three points.) Cross sections perpendicular to this base *and perpendicular to the base of* $\mathcal{S}$ are also semicircles with their own bases in $\mathcal{S}$. Find the volume of the solid. (Evaluate the integral in this one.)

6. (7 points) Find the average value of the function $y = \sin x$ on the interval $[0, \pi]$. (Evaluate completely to get an exact numerical answer.)

7. (7 points) State precisely the Mean Value Theorem for Integrals. (Completeness counts.)

8. (8/6/4 points) Evaluate each of the following integrals (I'd use integration by parts).

   (a) $\int x^2 e^{3x}\, dx$

   (b) $\int \arctan(3x)\, dx$

   (c) $\int \sqrt[3]{x}\, \ln x\, dx$

9. (8/6/4 points) Evaluate the following intervals. (These might be quickies if you remember certain formulas. Otherwise you need a trick or two.)

   (a) $\int \ln(5x)\, dx$

   (b) $\int \sec(5x)\, dx$

   (c) $\int \sec^3(5x)\, dx$

10. (8/6/4 points) Evaluate each of the following intervals. (I'd use some trig identities.)

    (a) $\int (1 + \cos 5x)^2\, dx$

    (b) $\int \tan^3 x\, dx$

    (c) $\int \sin^3 x\, \cos^3 x\, dx$

## $\star \quad \star \quad \star$ Extras $\star \quad \star \quad \star$

Feel free to do these on the back of the previous page or elsewhere. Just tell me where to look.

A. ($\star$) Find the exact area bounded by the $x$-axis, the $y$-axis, the line $x = 2$ and the curve $y = 1 + \sqrt{4 - x^2}$.

B. ($\star\star$) Same as in problem A, but with $x = 1$ instead of $x = 2$.

C. ($\star$) Consider the region whose area you found in problem #1. Now consider the solid formed by the union of all line segments drawn from points in this region to the point 5 units above the above the $xy$-plane, directly above the origin. Find the volume of this solid.

D. ($\star$) Use the method of shells to find the volume of the solid obtained by rotating the region given in problem #4 about the $x$-axis.

E. ($\star$) Prove the Mean Value Theorem for Integerals by applying the usual Mean Value Theorem (for derivatives) to the function $\int_a^x f(t)\,dt$.

F. ($\star$) Evaluate the integral $\int \cos mx\ \cos nx\,dx$, where $m$ and $n$ are constants.

G. ($\star$) Find a reduction formula for the integral $\int \sec^{2n-1} x\,dx$, where $n$ is a positive integer. The trick we used for $\int \sec^3 x\,dx$ works.

H. ($\star\star$) Here's a nice identity: $\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$.

  (a) Prove that the identity is true.
  (b) Use the techniques of integration we've been studying to evaluate $\int \sin^3 x\,dx$.
  (c) Use the identity to find this same integral.
  (d) use the equality of the integrals you found to write a similar identity for $\cos^3 x$.

I. ($\star\cdots\star$) Ask a question you wish I had asked and answer it. Points may vary. Offer void where prohibited by law.