Calc II — Fall 2007

MATH 222 — Name: ____________________

T1

Instructions: Write answers to problems *on separate paper.* You may NOT use calculators or any electronic devices or notes of any kind. Each st⋆rred problem is extra credit and each ⋆ is worth 5 points. (These are just more problems, but harder. They're worth fewer points so that you're not unduly tempted.) Loads of points are possible on the test, but the highest grade that I will award is 110 points.

Unless otherwise specified, you may leave **definite** integrals unevaluated ("just set it up"), but they must be "ready to evaluate," that is, each must be a definite integral involving one variable only (e.g., no mixed x's and y's) with **explicit functions and explicit limits of integration and no absolute values whatsoever**. However, if you would like to evaluate the integrals, they are worth an extra 4 points each. (*Warning:* Save these for last; some may be too difficult.)

Diminishing returns: Phrases such as "8/6/4 points" (see problem #4, for instance) refer to the points awarded for doing several parts of a problem. The example here indicates that 8 points will be awarded if any one problem of the three is done correctly, $8 + 6$ points if any two are correct, $8 + 6 + 4$ points for all three.

1. (10/6 points) Find the exact areas of the regions described below. (Do not merely write integrals in this one; evaluate completely to find the exact numerical result.)
   (a) the region bounded by the curves $x = 2y^2$ and $x + y = 1$.
   (b) the region bounded by the curves $y = \sin(\pi x/2)$ and $y = 2 - x$
2. (12 points — an "A" problem) Find the area bounded by $y = 2 - |x|$ and $y = x(x - 2)$. (Set it up.)
3. (12 points) Consider the region bounded by the x-axis, the lines $x = 1$ and $x = 2$, and the curve $y = 1/x^2$. A vertical line $x = b$ splits this region into two equal areas. Find the value of b.
4. (8/6/3/3 points) Use the method of disks/washers to find the volumes of the solids obtained by rotating the region bounded by the curves $x = 2y$ and $y = \sqrt{x}$ about each of the following lines. (Set up the integrals.)
   (a) the x-axis
   (b) the y-axis
   (c) the line $y = 2$
   (d) the line $x = -1$

5. (8/6/3/3 points) Use the method of shells to find the volumes of the solids obtained by rotating the region bounded by the curves $y = x^2$ and $y = 2 - x$ about each of the following lines. (Set up the integrals.)

   (a) the x-axis

   (b) the y-axis

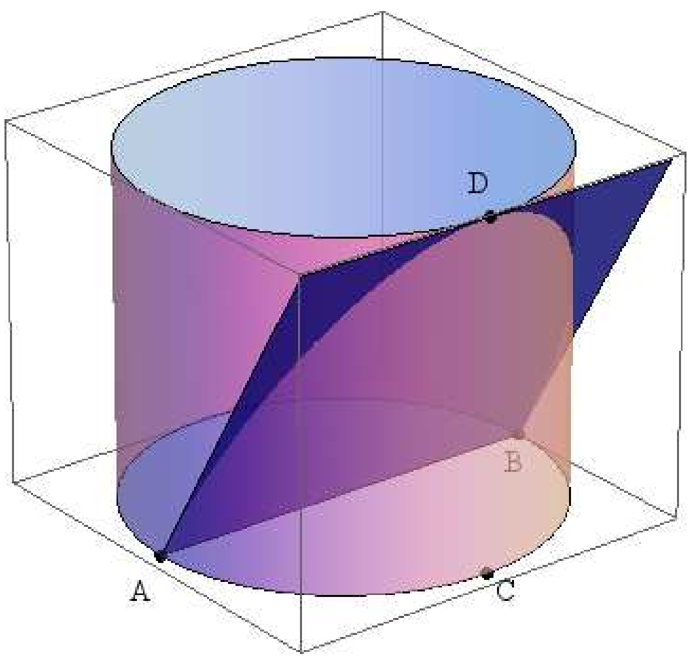
   (c) the line $x = 2$

   (d) the line $y = 0$

6. (12 points) A solid's base is the region in the xy-plane bounded by the semicircle $y = \sqrt{R^2 - x^2}$ and the line $y = 0$. Cross sections perpendicular to the x-axis (and perpendicular to the base of the solid, of course) are squares. Find the volume of the solid. (Evaluate the integral in this one.)

($\star$) Repeat the previous problem, but with the cross-sections perpendicular to the y-axis instead.

7. (15 points — an "A+" problem) This is a modified example from the text. Consider a right circular cylinder of radius R and height H. A "wedge" of the cylinder is formed as follows. Set the cylinder on one of its circular bases, so that this base is centered at the origin of xy-plane and the rest of the cylinder lies above the plane. The points $A = (0, -R)$, $B = (0, R)$ and $C = (R, 0)$ lie on the circular base; consider the point D lying above the xy-plane, H units above the point C. (The point D is on the "top" of the cylinder.) The plane formed by A,B and D cuts the cylinder. Find the volume of the wedge of the cylinder lying below this cutting plane and above the xy-plane (and within the cylinder).



8. (8 points) Find the average value of the function $y = 1/(3x+4)$ over the interval $[-1, 1]$. (Evaluate completely to get an exact numerical answer.)

9. (5 points) State precisely the Mean Value Theorem for Integrals.

10. (8/6/4 points) Evaluate each of the following integrals.

    (a) $\displaystyle\int (x+2)e^{5x}\,dx$

    (b) $\displaystyle\int \arcsin(3x)\,dx$

    (c) $\displaystyle\int \frac{\ln x}{\sqrt[3]{x}}\,dx$

11. (8/6/4 points) Evaluate each of the following integrals.

    (a) $\displaystyle\int x^2\cos(5x)\,dx$

    (b) $\displaystyle\int x\arctan x\,dx$

    (c) $\displaystyle\int e^{5x}\cos 2x\,dx$

12. (6 points) Integrate. $\displaystyle\int \frac{1}{\cos x}dx$

⋆ ⋆ ⋆ Extras ⋆ ⋆ ⋆

Feel free to do these on the back of the previous page or elsewhere. Just tell me where to look.

A. (⋆) Consider the region in the xy-plane bounded by the curve $y = x^2$ and the line $y = 4$. Now consider the solid formed by the union of all line segments drawn from points in this region to the point 5 units above the xy-plane and directly over the origin. Find the volume of this solid.

B. (⋆) Prove the Mean Value Theorem for Integrals by applying the usual Mean Value Theorem (for derivatives) to the function $\int_a^x f(t)\,dt$. Explain each step.

C. (⋆) Find the mean value c guaranteed by the Mean Value Theorem for Integrals in problem #8.

D. (⋆⋆) A spherical tank 10 meters in diameter and filled with liquid. Find the work done by pumping all the fluid out of the tank and up to a height 5 meters above the top of the tank. Use ρ for the density of the liquid and g for the acceleration due to gravity.

E. (⋆) Evaluate the integral $\int \cos mx\ \cos nx\,dx$, where m and n are nonzero constants.

F. (⋆) Find a reduction formula for the integral $\int \sin^n x\,dx$, where n is a positive integer.

G. (⋆ · · · ⋆) Surely I forgot something you were ready for. Ask a question you wish I had asked and answer it. Points may vary. Offer void where prohibited by law.