Calc II
MATH 222

Spring 2009
Name:

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T1

Instructions: Write answers to problems *on separate paper.* You may NOT use calculators or any electronic devices or notes of any kind. Each st⋆rred problem is extra credit and each ⋆ is worth 5 points. (These are just more problems, but harder. They're worth fewer points so that you're not unduly tempted.) Loads of points are possible on the test, but the highest grade that I will award is 115 points.

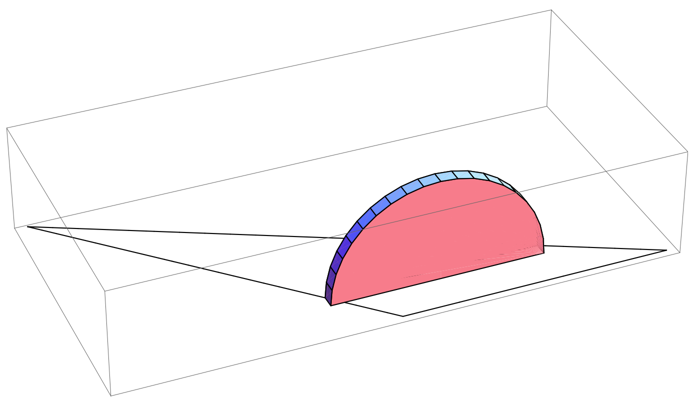
Unless otherwise specified, leave **definite** integrals unevaluated ("just set it up") but "ready to evaluate." That is, a "set-up" integral must be a definite integral involving one variable only (e.g., no mixed x's and y's) with **explicit functions and explicit limits of integration and no absolute values whatsoever**.

Diminishing returns: Phrases such as "7/5/3 points" (see problem #6, for instance) refer to the points awarded for doing several parts of a problem. The example here indicates that 7 points will be awarded if any one problem of the three is done correctly, $7 + 5$ points if any two are correct, $7 + 5 + 3$ points for all three.

1. (12 points) Find the exact area of the region bounded by the curves $y = x^2$ and $y = 3 - 2x$. Evaluate completely and simplify.

2. (10 points) Find the area bounded by $y = 2|x|$ and $y = -x^2 + 4x + 2$. (Just set up the integral(s). You might need the quadratic formula to find the limits.)

3. (10 points) A solid $\mathcal{S}$ has as its base the region $\mathcal{R}$ in the xy-plane bounded by the lines $y = 2x$, $y = x$ and $x = 2$. Cross sections of $\mathcal{S}$ perpendicular to the x-axis (and perpendicular to the base of $\mathcal{S}$, of course) are semicircles with bases in $\mathcal{R}$. Use integration to the volume of the solid. (Evaluate and simplify.)



4. (10 points) Use the method of disks/washers to find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y = x^3$ about the x-axis. (Evaluate and simplify.)

5. (10 points) Use the method of shells to find the volume of the solid obtained by rotating the region bounded by the curves $y = 1 - x^2$ and $y = (x-1)^2$ about the y-axis. (Set up the integral(s).)

6. (7/5/3 points) Find the volumes of the solids obtained by rotating the region described in problem #4 about the following lines. (Set up the integral(s).)

   (a) the y-axis

   (b) the line $y = 2$

   (c) the line $x = -3$

7. (7/5/3 points) Find the volumes of the solids obtained by rotating the region described in problem #5 about the following lines. (Set up the integral(s).)

   (a) the x-axis

   (b) the line $y = 2$

   (c) the line $x = -3$

8. (5 points) You probably realized that using disks/washers for problem #5 would be more difficult than using shells. Convince yourself that this is true by actually using disks/washers to find the volume! (Set up the integral(s).)

9. (10 points) Find the average value of the function $y = \dfrac{1}{x-1}$ over the interval $[2, 6]$. (Evaluate completely and simplify.)

10. (5 points) State precisely the Mean Value Theorem for Integrals. (Completeness counts.)

11. (10/7/4 points) Evaluate each of the following integrals (I'd use integration by parts).

    (a) $\int x^2 \cos(5x)\, dx$

    (b) $\int \arcsin(2x)\, dx$

    (c) $\displaystyle\int \frac{\ln x}{\sqrt{x}}\, dx$

12. (10/7/4 points) Evaluate each of the following integrals. (I'd use some trig identities.)

    (a) $\int (1 - \cos 3x)^2\, dx$

    (b) $\int \sin^2 x\, \cos^5 x\, dx$

    (c) $\int \sec^4(5x) \tan^3(5x)\, dx$

$\star \quad \star \quad \star$ Extras $\star \quad \star \quad \star$

Feel free to do these on the back of the previous page or elsewhere. Just tell me where to look.

A. ($\star$) In problem #3, find the volume of the resulting solid if the semi-circular cross-sections are perpendicular the y-axis instead of the x-axis (with the same $\mathcal{R}$ region for the base).

B. ($\star$) Consider the region $\mathcal{A}$ whose area you found in problem #1. Let P denote the point 5 units above the xy-plane, directly above $(0,0)$. (This is the point $(0,0,5)$ in xyz-coordinates, something we introduce in Calc III.) Now consider the solid formed by the union of all line segments drawn from points in $\mathcal{A}$ to the point P. Find the volume of this solid.

C. ($\star$) A spherical tank of radius R is filled with liquid. Find the work done by pumping all the liquid out of the tank up to a height H meters above the top of the tank. Use ρ for the density of the liquid and g for the accelearation due to gravity.

D. ($\star$) Evaluate $\displaystyle\int x \arctan x \, dx$.

E. ($\star$) This follows a discussion in class yesterday.

(a) Evaluate $\displaystyle\int \frac{1}{1-\sin x}\, dx$ using the "conjugation" trick.

(b) Evaluate $\displaystyle\int \frac{1}{1-\cos x}\, dx$ using a different method—let $x = 2u$ and exploit a ubiquitous double-angle identity.

F. ($\star$) Prove the Mean Value Theorem for Integerals by applying the usual Mean Value Theorem (for derivatives) to the function $\int_a^x f(t)\, dt$.

G. ($\star$) Evaluate the integral $\int \sin mx \, \sin nx \, dx$, where m and n are constants with $m \neq n$.

H. ($\star$) Find a reduction formula for the integral $\int \sin^n x \, dx$, where n is a positive integer.

I. ($\star\star$) Here's a nice identity: $\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$.

(a) Prove that the identity is true using familiar trig identities.

(b) Use the techniques of integration we've been studying to evaluate $\int \sin^3 x \, dx$.

(c) Use the identity to find this same integral.

(d) Use the equality of your results to find an identity for $\cos^3 x$. (Beware of constants! But consider what must happen when $x = 0$, say.)

J. ($\star \cdots \star$) Surely I forgot something you were ready for. Ask a question you wish I had asked and answer it. Points will vary. (Trivial questions or repeats get few, if any, points.)