Calc II

MATH 222

Fall 2005

Name: ______________

T2

Instructions: Write answers to problems *on separate paper.* You may NOT use calculators or any electronic devices or notes of any kind. Each st$\star$rred problem is extra credit and each $\star$ is worth 5 points. (These are just more problems, but harder. They're worth fewer points so that you're not unduly tempted.) Loads of points are possible on the test, but the highest grade that I will award is 115 points.

In specified cases you may leave **definite** integrals unevaluated (I'll say, "just set it up"), but these must then be "ready to evaluate," that is, each must be a definite integral involving one variable only (e.g., no mixed x's and y's) with **explicit functions and explicit limits of integration and no absolute values whatsoever**. However, if you would like to evaluate the integrals, they are worth an extra 4 points each. (*Warning:* Save these for last; some may be too difficult or impossible.)

Diminishing returns: Phrases such as "8/6/4 points" refer to the points awarded for doing several parts of a problem. The example here indicates that 8 points will be awarded if any one problem of the three is done correctly, $8+6$ points if any two are correct, $8+6+4$ points for all three.

1. (10 points) Evaluate the integral $\displaystyle\int \sqrt{9-4x^2}\,dx$.

2. (10/8/6 points) Evaluate the integrals.

   (a) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-4}}$

   (b) $\displaystyle\int \frac{dx}{(x^2+2x+2)^2}$

   (c) $\displaystyle\int \frac{dx}{\sqrt{5-3x^2}}$

3. (10 points) Evaluate the integral $\displaystyle\int \frac{dx}{x^3-4x}$.

4. (12/8/5 points) Evaluate the integrals.

   (a) $\displaystyle\int \frac{x^3-2x+1}{(x+1)(x+2)}\,dx$

   (b) $\displaystyle\int \frac{x^2}{(x+1)^2(x+2)}\,dx$

   (c) $\displaystyle\int \frac{x^3}{(x^2+1)^2}\,dx$

5. (5/5/3/3/3 points) Evaluate each definite integral completely and simplify. If the integral diverges, say so and state how it diverges (e.g., to ∞ or $-\infty$).

   (a) $\displaystyle\int_0^1 \frac{dx}{x^2}$

   (b) $\displaystyle\int_8^\infty \frac{dx}{x^{8/3}}$

   (c) $\displaystyle\int_{-1}^8 \frac{dx}{x^{8/3}}$

   (d) $\displaystyle\int_{-3}^0 \frac{dx}{x^2-4}$

   (e) $\displaystyle\int_{-\infty}^0 \frac{e^x\,dx}{e^{2x}+1}$

6. (10 points) Calculate the arc length of the curve $y = \ln x$ from $x = 1$ to $x = e$. (Set it up.)

7. (12/8/4/4/4/4 points) Consider the portion of the curve $y = \sin x$, $0 \le x \le \pi$. Calculate the surface areas of each solid of revolution, where the arc is rotated about each of the following axes. (Set them up.)

   (a) the y-axis

   (b) the x-axis (be careful)

   (c) the line $y = 2$

   (d) the line $x = -2$ (keep being careful)

   (e) the line $x = \pi/2$ (amusing but a bit tricky)

   (f) the line $y = 1/2$ (seriously)

8. (6 points) Use the Comparison Theorem to decide if the integral

$$\int_1^\infty \frac{1+\cos(\sqrt{x})}{e^x}\,dx$$

is convergent or divergent. *Your answer will be worth very little without a clear explanation.*

⋆ ⋆ ⋆ Extras ⋆ ⋆ ⋆

Feel free to do these on the back of the previous page or elsewhere. Just tell me where to look.

A. ($\star$) Use the Weierstrass substitution to transform the integral $\displaystyle\int \frac{2\sin x+1}{2\sin x+\cos x}\,dx$ into an integral of a rational function. (Don't evaluate the resulting integral.)

B. ($\star$) Calculate the arc length of the perimeter of the region bounded by the two curves $y = x^3$ and $y = x^2 - 3x$. (Set it up. Show no fear.)

C. ($\star$) The "Horn of Gabriel" is the solid of revolution obtained by rotating the curve $y = 1/x$ about the x-axis for $x > 1$, i.e., on the interval $x \in [1, \infty)$. Show that the volume of this solid is finite but that its surface area is infinite (calculate both). (This "paradox" is supposed to freak you out, so don't think about the implications until later.)

D. ($\star$) You'd "set up" a partial fraction decomposition for the function $\dfrac{1}{x(x+2)}$ by writing

$$\frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

and solving for the unknown constants A and B. How would you set up the partial fraction decomposition of the following function?

$$\frac{x^5 - x^4}{(x+1)^3(x^2+2)^2}$$

E. ($\star$) The standard procedure for decomposing the function $\dfrac{x^3+2}{(x-1)(x+2)}$ using partial fractions would be to first do a long division and to then decompose the "remainder" using methods alluded to above. But it is clear that the result will be that

$$\frac{x^3+2}{(x-1)(x+2)} = Ax + B + \frac{C}{x+1} + \frac{D}{x-2},$$

for some constants A, B, C, D. Instead of using long division, use the tricks that we have typically applied to the remainder term to solve for these constants. (*Start* by multiplying in the usual way, to clear the denominators, then choosing convenient values for x ...)

F. ($\star$) Refer to problem #5e and consider the integral $\displaystyle\int_{-\infty}^{\infty} \frac{e^x\,dx}{e^{2x}+1}$, where the integrand hasn't changed but the limits of integration are now $-\infty$ to ∞. Show that

$$\int_{-\infty}^{\infty} \frac{e^x\,dx}{e^{2x}+1} = 2\int_{-\infty}^{0} \frac{e^x\,dx}{e^{2x}+1}.$$

(Can symmetry be used? Explain fully.)

G. ($\star\star$) Stare 'em down! Without evaluating the integrals, decide which of the following are convergent (finite) and which are divergent. *No credit will be given without a short, compleiing explanation.*

(a) $\displaystyle\int_1^{\infty} \frac{2-\sqrt{2}\cos x}{e^x}\,dx$

(b) $\displaystyle\int_2^{\infty} \frac{1}{e^{2x}-x}\,dx$

(c) $\displaystyle\int_3^{\infty} \frac{x}{e^{3x}}\,dx$

(d) $\displaystyle\int_4^{\infty} \frac{x^3+3x+1}{x^4+2x+2}\,dx$

(e) $\displaystyle\int_5^{\infty} \frac{x^2+2x+1}{x^4+2x+2}\,dx$

H. ($\star\cdots\star$) Ask a question you wish I had asked and answer it. Points may vary. Offer void where prohibited by law.