

T2

Instructions: Write answers to problems *on separate paper*. You may NOT use calculators or any electronic devices or notes of any kind. Each starred problem is extra credit and each \star is worth 5 points. (These are just more problems, but harder. They're worth fewer points so that you're not unduly tempted.) Loads of points are possible on the test, but the highest grade that I will award is 115 points.

Diminishing returns: Phrases such as "8/6/4 points" refer to the points awarded for doing several parts of a problem. The example here indicates that 8 points will be awarded if **any** one problem of three is done correctly, 8 + 6 points if any two are correct, 8 + 6 + 4 points for all three.

1. (10/10/8/8 points) Evaluate each of the following integrals.

(a) $\int \sin^5 x \cos^3 x \, dx$

(b) $\int (2 - \sin 5x)^2 \, dx$

(c) $\int \sec^3 x \tan^3 x \, dx$

(d) $\int \sec^4 x \tan^3 x \, dx$

2. (10/10/8 points each) Evaluate the integrals.

(a) $\int \frac{dx}{\sqrt{9-x^2}}$

(b) $\int_0^4 \frac{dx}{\sqrt{9+x^2}}$

(c) $\int \frac{dx}{9+4x^2}$

3. (3/3/2/2 points) “Partially” set up partial fraction decompositions for each of the following by writing the function as a sum of simple fractions with *undetermined coefficients*. As an example of what I mean, if asked to set up the decomposition of the function $\frac{2x - 3}{(x - 3)(x + 1)}$, you would answer

$$= \frac{A}{x - 3} + \frac{B}{x + 1}.$$

(Here, A and B are the coefficients to be determined, but you are not determining them in the next four problems.) Do likewise for each of the following.

(a) $\frac{2x + 3}{x^2(x - 4)}$

(b) $\frac{2x + 3}{x(x^2 + 4)}$

(c) $\frac{2x + 3}{(x + 1)(x^2 + 4)^2}$

(d) $\frac{2x^2 + 3}{(x - 3)(x + 2)}$ (trickier, but can be answered in like fashion)

4. (10/8/8 points) Evaluate the integrals.

(a) $\int \frac{2x - 3}{(x - 3)(x + 1)} dx$

(b) $\int \frac{5x^2 - 4x + 4}{(x - 2)x^2} dx$

(c) $\int \frac{x(x + 3)}{(x - 2)(x^2 + 1)} dx$

5. (10/8/8/6 points) Evaluate the integrals.

(a) $\int \frac{2x^3 - x^2 - 6x + 1}{(x-1)(x+1)} dx$

(b) $\int \cot x \ln(\sin x) dx$

(c) $\int_2^5 \frac{x(x-2)}{\sqrt{x-1}} dx$

(d) $\int \frac{1}{x + \sqrt[5]{x}} dx$ [One extra \star for using $\sqrt[n]{x}$ in place of $\sqrt[5]{x}$ (if done correctly $\circ\triangle\circ$).]

$\star \quad \star \quad \star$ Extras $\star \quad \star \quad \star$

A. (\star) Integrate. $\int \frac{dx}{\sqrt{x} + \sqrt{x+1}}$

B. (\star) Integrate. $\int \frac{dx}{x + \sqrt{x^2 + 2x}}$

C. (\star) Integrate by using a hyperbolic trig substitution. $\int \frac{dx}{\sqrt{x^2 + 1}}$

D. (\star) Sketch beautiful graphs of $y = \sinh x$ and $y = \cosh x$ on the same set of axes.

E. (\star) Integrate. $\int \sin(7x) \cos(5x) dx$ (Hint: consider product-to-sum formulas.)

F. (\star) Use the “method of undetermined coefficients” (rather than integration by parts FOUR TIMES) to evaluate $\int x^4 e^x dx$.

G. ($\star \cdots \star$) Ask a question you wish I had asked and answer it. Points may vary. Offer void where prohibited by law.