

T2

**Instructions:** Write answers to problems *on separate paper*. You may NOT use calculators or any electronic devices or notes of any kind. Each starred problem is extra credit and each  $\star$  is worth 5 points. (These are just more problems, but harder. They're worth fewer points so that you're not unduly tempted.) Loads of points are possible on the test, but the highest grade that I will award is 110 points.

**Diminishing returns:** Phrases such as "8/6/4 points" refer to the points awarded for doing several problems in a set. The example here indicates that 8 points will be awarded if **any** one problem of the three is done correctly, 8 + 6 points if any two are correct, 8 + 6 + 4 points for all three. The problems are not typically listed in order of increasing or decreasing difficulty.

1. (20/12/8 points) Evaluate the integrals.

(a)  $\int \frac{dx}{x\sqrt{4x^2 - 1}}$

(b)  $\int \frac{x + 3}{\sqrt{2x - x^2}}$

(c)  $\int \frac{dx}{7 + x^2}$

2. (8/5/4/3 points) Below you will "partially" set up some partial fraction decompositions by writing a rational function as a sum of simpler fractions with *undetermined coefficients*. As an example of what I mean, if asked to partially set up the decomposition of the function  $\frac{2x - 3}{(x - 3)(x + 1)}$ , you would answer

$$" = \frac{A}{x - 3} + \frac{B}{x + 1}."$$

(Here,  $A$  and  $B$  are the coefficients to be determined, but you are not being asked to determine them in this set of problems.) Partially set up partial fraction decompositions for each of the following.

(a)  $\frac{2x + 3}{x^2(x - 4)}$

(b)  $\frac{2x + 3}{x(x^2 + 4)}$

(c)  $\frac{2x + 3}{(x + 1)(x^2 + 4)^2}$

(d)  $\frac{2x^2 + 3}{(x - 3)(x + 2)}$

(This one is trickier, but can be answered in like fashion.)

3. (20/12/8 points) Evaluate the integrals

(a)  $\int \frac{x+2}{x^2-1} dx.$

(b)  $\int \frac{x+2}{x^3-x} dx.$

(c)  $\int \frac{2}{x(x^2+1)} dx.$

4. (10/8/6 points each) Evaluate each definite integral completely and simplify if the integral converges. If the integral diverges, say so and state how it diverges (e.g., to  $\infty$  or  $-\infty$ ). (Very little credit will be given for answers with incorrect reasoning or no work.)

(a)  $\int_3^\infty \frac{dx}{x^2}$

(b)  $\int_{-1}^8 \frac{dx}{x^{2/3}}$

(c)  $\int_0^4 \frac{x}{(x-2)^2} dx$

5. (8/6 points) Use the Comparison Theorem to decide if the integrals below are convergent or divergent. (Answers will be worth very little without clear and correct explanations.)

(a)  $\int_1^\infty \frac{x^2}{2x^4+1} dx$

(b)  $\int_1^\infty \frac{x^3}{2x^4-1} dx$

★ ★ ★ ★ ★ Extras ★ ★ ★ ★ ★

A. (★) Integrate and simplify  $\int \frac{1}{x + \sqrt[3]{x}} dx$

B. (★) Integrate.  $\int \frac{dx}{\sqrt{x+1} + \sqrt{x}}$

C. (★) Carefully explain whether the integral

$$\int_1^\infty \frac{x^2}{2x^4-1} dx$$

is convergent or divergent. (An answer will be worth very little without a clear explanation.)

D. (★) Precisely state the Trapezoid Rule for numerical integration.

E. (★⋯★) Ask a question you wish I had asked and answer it. Points may vary. Offer void where prohibited by law.