Calc II | Fall 2007
MATH 222 | Name: ____________

T3

Instructions: Write answers to problems *on separate paper.* You may NOT use calculators or any electronic devices or notes of any kind. Each st⋆rred problem is extra credit and each ⋆ is worth 5 points. Loads of points are possible on the test, but the highest grade that I will award is 115 points.

1. (10/8/6/4 points) Evaluate each definite integral completely and simplify. If the integral diverges, say so and state how it diverges (e.g., to ∞ or $-\infty$). (Very little credit will be given for answers with no work or incorrect reasoning.)
   (a) $\displaystyle\int_{-1}^{1} \frac{dx}{x^3}$
   (b) $\displaystyle\int_{0}^{1} \frac{dx}{x^{1/3}}$
   (c) $\displaystyle\int_{0}^{2} \frac{1}{x-2}\,dx$
   (d) $\displaystyle\int_{-1}^{\infty} \frac{dx}{x^2+1}$
2. (8 points) Calculate the arc length of the curve $y = \sin x$ from $x = 0$ to $x = \pi$. (Set it up.)
3. (8/6/3/3 points) The portion of the curve $y = \sin x$, $0 \le x \le \pi$ is rotated about each of the following axes. Calculate the areas of each surface of revolution formed. (Set up the integrals.)
   (a) the y-axis
   (b) the x-axis
   (c) the line $y = -1$
   (d) the line $x = -2\pi$
4. (8 points) Use the Comparison Theorem to decide if the integral
$$\int_{1}^{\infty} \frac{1}{x^3 + \sqrt[3]{x}}\,dx$$
is convergent or divergent. (An answer will be worth very little without a clear explanation.)
5. (12 points) Find the centroid $(\bar{x}, \bar{y})$ of the region below the curve $y = \sin x$, between $x = 0$ and $x = \pi$. (Don't merely set this one up — calculate it completely and simplify.)
6. (5 points) State the definition of a convergent sequence. (I'll get you started: "The sequence (a_n) converges to a limit L if and only if for every $\varepsilon > 0$, ...")

7. (5 points) State (the hypotheses and conclusions of) the monotone convergence theorem.

8. (6 points) Decide (and show) whether the sequence $\left\{\dfrac{n+4}{n^2}\right\}_{n=1}^{\infty}$ is increasing or decreasing.

9. (3 points each) Determine whether each of the following sequences $\{a_n\}$ converges or diverges for the given a_n. If it converges, find its limit. If it diverges to $+\infty$, say so. If it diverges to $-\infty$, say so. If it diverges in some other way, say how. No credit for "diverges" or "converges", but no penalties for incorrect answers.

   (a) $a_n = \dfrac{n^2+1}{2n+3n^2}$

   (b) $a_n = (-1)^n \dfrac{n^2+1}{2n+3n^2}$

   (c) $a_n = (-1)^n \left(\dfrac{n^2+1}{2n^3+3n}\right)$

   (d) $a_n = \dfrac{3n^2}{5^n}$

   (e) $a_n = \dfrac{1}{\ln(\sqrt{n}+1)}$

   (f) $a_n = \tan\left(\dfrac{1}{n}\right)$

   (g) $a_n = n\dfrac{n!}{(n+1)!}$

   (h) $a_n = \dfrac{(n+1)\sqrt{9n^2+1}}{2n^2}$

   (i) $a_n = \sin\dfrac{(-1)^n}{n}$

   (j) $a_n = \dfrac{n^n}{5^n}$

⋆ ⋆ ⋆ Extras ⋆ ⋆ ⋆

Feel free to do these on the back of the previous page or elsewhere. Just tell me where to look.

A. ($\star$) Use Pappus's theorem to find the volume of the solid obtained by rotating a triangle about the line $x = -1$ when the vertices of the triangle are the points $(-1, -1)$, $(1, 2)$ and $(4, 0)$. (There are other famous theorems of Pappus of Alexandria, who lived from about 290 to 350 AD.)

B. ($\star$) The disk of radius 1 centered at $(0, 2)$ is removed from the disk of radius 4 centered at $(0, 0)$. Find the centroid of the resulting figure.

C. ($\star$) Find the centroid of the *solid* cone of radius R and height H.

D. ($\star\star$) Use the Comparison Theorem to decide if the integrals below are convergent or divergent. (These differ slightly from problem #4.) Justify your answers.

(a) $\displaystyle\int_2^\infty \frac{1}{x^3 - \sqrt[3]{x}}\, dx$

(b) $\displaystyle\int_0^\infty \frac{1}{x^3 + \sqrt[3]{x}}\, dx$

(c) $\displaystyle\int_1^\infty \frac{1}{x^3 - \sqrt[3]{x}}\, dx$

E. ($\star$) Show that the sequence $\displaystyle\left\{\frac{n^3}{2^n}\right\}_{n=1}^{\infty}$ is *eventually* decreasing. (I'd look at the ratio a_{n+1}/a_n and try to show that it is eventually less than 1.)

F. ($\star\star$) Determine whether each of the following sequences $\{a_n\}$ converges or diverges for the given a_n. If it converges, find its limit. If it diverges to $+\infty$, say so. If it diverges to $-\infty$, say so. If it diverges in some other way, say how. No credit for "diverges" or "converges", but no penalties for incorrect answers.

(a) $a_n = (2^n + 3^n)^{4/n}$.

(b) $a_n = \dfrac{\ln(5n^2)}{\ln(3n^5)}$

(c) $a_n = \left(1 + \dfrac{5}{n}\right)^{3n}$

G. ($\star\cdots\star$) Ask a question you wish I had asked and answer it. Points may vary. Offer void where prohibited by law.