

A surface, concave up at a point in the x and y directions, but with no min or max.

Let $x = r \cos \theta, y = r \sin \theta, z = r^2 \cos(4\theta)$. Then $\partial z / \partial x = 0, \partial z / \partial y = 0, \partial^2 z / \partial x^2 > 0$ and $\partial^2 z / \partial y^2 > 0$ at the point $(x, y) = (0, 0)$. (This is a terrific [and subtle] exercise in using the chain rule.) But there is no min or max at $(0, 0)$.

ParametricPlot3D [$x = \sqrt{rr} \text{Cos}[th]; y = \sqrt{rr} \text{Sin}[th];$

$\{x, y, rr \text{Cos}[4 th]\}, \{rr, 0, 1/3\}, \{th, 0, 2\pi\}$]

