Calc III — Spring 2006

MATH 223 — Name: ____________________

Test #1

Instructions: Answer all problems correctly. Calculators are not allowed. Each numbered problem is worth 8 points. Each st⋆rred problem is "extra credit" and each ⋆ is also worth 8 points. Do as many problems as you wish, but no grade higher than 115 points (including a curve, if any) will be awarded on the test.

In all problems on this test, P, Q and R will always refer to the same points as given in problem #1.

1. Consider the three points $P = (1, 1, 2)$, $Q = (1, 0, 3)$ and $R = (0, 0, 4)$. Find the vector $\mathbf{PR}$. Find the distance between points P and Q.
2. What point lies $2/3$ the way from the point P to the point R?
3. A sphere is centered at point R and passes through P. Write an equation for the sphere.
4. What is the angle at vertex P in the triangle formed from the points P, Q and R? (You may have to leave your answer in "calculator-ready form".)
5. Find a vector that bisects the angle at vertex P of the triangle PQR.
6. Find the area of the triangle PQR.
7. Write an equation in standard form for the plane containing triangle PQR. Next, write an equation for the plane parallel to this plane but passing through the point $S = (3, 4, 5)$.
8. Find parametric equations and the symmetric equations for the line passing through the points P and R.
9. Find the projection of the $\mathbf{PQ}$ onto the vector $\mathbf{PR}$.
10. Find the distance from the point Q to the line passing through P and R.

(⋆) While you're at it, find the projection of the *point* Q onto the line passing through P and R.

11. Write an equation for the plane containing the point Q and perpendicular to the line passing through P and R.
12. Find *parametric equations* for the plane containing P,Q,R.
13. Write a parameterization for the *line segment* joining points P and R, but such that this segment contains P and does not contain R. (You could call this a "half-open" line segment.)
14. Let L denote the line containing the origin and the point S from problem #7. At what point does the line L intersect the plane containing P,Q,R?
15. Use vectors to prove that the diagonals of a parallelogram always bisect each other.

16. Find the coordinates of the point obtained by rotating P about the positive z-axis through $40°$.

17. Find the coordinates of the centroid of triangle PQR.

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A. ($\star$) Find the coordinates of the point obtained by rotating P about the positive y-axis through $40°$.

B. ($\star$) Find the distance between the two planes given in problem #7.

C. ($\star$) Prove that the area of a simple plane quadrilateral $ABCD$ is given by $(1/2)||\mathbf{AC} \times \mathbf{BD}||$.

D. ($\star$) Are any of the angles in triangle PQR obtuse? Justify your answer.

E. ($\star$) Find the volume of the tetrahedron formed by the points P, Q, R and the origin.

F. ($\star\star$) The sphere of radius 5 centered at the origin intersects the plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ to form a circle. What is the radius of that circle?

G. ($\star\star$) Find the reflection of the origin through the plane formed by the points P, Q, R.

H. ($\star\star$) Consider the situation of two skew lines: L_1 parameterized as $P_1 + s\mathbf{v_1}$ and L_2 parameterized as $P_2 + t\mathbf{v_2}$, $s, t \in \mathbb{R}$. Find a formula for the distance between the two lines.

I. ($\star \cdots \star$) Ask a question you wish I had asked and answer it. Points vary depending on the difficulty of the question (and the correctness of the solution). Very few points (if any) will be awarded for a problem that is essentially represented elsewhere on the test. (In other words, no repeats.)