Calc III — Spring 2013

MATH 223 — Name: ______________

Test #1

Instructions: Answer all problems correctly. Write all responses *on separate paper* (provided) and keep the problems in their numbered order.

No calculators or other electronic gadgets may be used. However, you needn't spend much time simplifying answers.

Each numbered problem is worth 8 points. Each st$\star$rred problem is "extra credit" and each $\star$ is also worth 8 points. Do as many problems as you wish, but no grade higher than 110 points (including a curve, if any) will be awarded on the test.

In all problems, O will refer to the origin, and P, Q and R will always refer to these same points:

$$P = (1, -1, 2),\ Q = (2, 1, 0) \textit{ and } R = (-1, 5, 0).$$

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1. Find the vector **PR**. Find the distance between points P and Q.
2. What point lies 4/5 the way from the point P to the point R?
3. A sphere is centered at point P and passes through Q. Write an equation for the sphere.
4. Find the point S that makes $PQRS$ (in that order) a parallelogram.
5. What is the angle at vertex Q in the triangle formed from the points P, Q and R? (Leave your answer in "calculator-ready form".)
6. Find the area of the triangle PQR. Find the volume of tetrahedron $OPQR$.
7. Write an equation in standard form for
   (a) the plane containing triangle PQR, and for
   (b) the plane parallel to the plane in the previous part, but passing through the point $(0, 0, 10)$.
8. (a) Find parametric equations and symmetric equations for the line passing through the points P and R.
   (b) Write symmetric equations for the line passing through the points Q and R.
9. Find the projection of the vector **PR** onto the vector **PQ**.
10. Find the distance from the point R to the line passing through P and Q.

($\star$) While you're at it, find the projection of the *point* R onto the line passing through P and Q.

($\star$) You may as well find the reflection of the point R with respect to the line passing through P and Q.

11. Find *parametric equations* for the plane containing P,Q,R. (You can use s and t for the two parameters needed.)

12. Write a parameterization for the *line segment* $\overline{PQ}$ joining points P and Q. (Many people leave out a crucial part of the answer — don't let that happen to you!)

13. At what point does the line given by $\dfrac{x-2}{3} = \dfrac{1-y}{2} = \dfrac{2z-3}{4}$ intersect the plane $x+2y-z=0$?

14. Find an equation for the plane that is perpendicular to the plane containing triangle PQR and that contains the line through P and Q.

15. Consider the two points $K_1 = (1,-1,4)$ and $K_2 = (1,2,0)$. Prove that K_1 and K_2 lie on opposite sides of the plane containing P,Q,R. (See problems $\star$G and $\star$H for related, but successively greater challenges.)

$\star \quad \star \quad \star \quad \star \quad \star$

A. ($\star$) Write a parameterization for the triangle PQR. (You might think about problem #11.)

B. ($\star$) Find the distance between the two planes given in problem #7a.

C. ($\star$) Are any of the angles in triangle PQR obtuse? Justify your answer.

D. ($\star$) Find the line that bisects the angle at vertex Q of the triangle PQR.

E. ($\star$) The sphere of radius 3 centered at $(0,0,1)$ intersects the plane $x+2y+3z=0$ to form a circle. What is the radius of that circle?

F. ($\star$) Find the distance between the lines parameterized by

$$(0,1,-1) + t\langle 1,1,2\rangle \quad \text{and} \quad (2,0,1) + s\langle 0,-1,1\rangle.$$

G. ($\star$) Prove that the line segment joining K_1 and K_2 in problem #15 actually passes through the parallelogram $PQRS$. (See problem #4 for the definition of S. Notice that this is a stronger statement than the one given in problem #15. If you prove this one, you've automatically solved that one.)

H. ($\star\star$) Does the line segment joining K_1 and K_2 in problem #15 actually passes through the triangle PQR? (An affirmative answer would imply the results of both problems #15 and $\star$G.)

I. ($\star\cdots\star$) Surely you were ready for something I didn't ask! Ask a question you wish I had asked and answer it. Points vary depending on the difficulty of the question (and the correctness of the solution). Very few points (if any) will be awarded for a problem that is essentially represented elsewhere on the test. (In other words, no repeats.)