Calc III
MATH 223

Spring 2013
Name: ______________

Test #2

Instructions: Answer all problems correctly. Each numbered problem is worth $\boxed{\text{12 points}}$ unless otherwise specified. Each st$\star$rred problem is "extra credit" and each $\star$ is worth 8 points. Do as many problems as you wish, but no grade higher than 110 points (including a curve, if any) will be awarded on the test.

You may leave unevaluated any integrals that need to be calculated in the course of solving these problems. In other cases, you may leave answers in unsimplified, "calculator-ready form" — you needn't rationalize denominators or do other trivial but time-consuming simplifications in your answers — unless some aspect of the answer needs simplification to be clear or unless otherwise specified. Calculators and electronic devices are not allowed.

1. (2 points each) Choose exactly five of the following equations and identify by name the surfaces described (e.g., ellipsoid, cone, paraboloid, etc.).

   (a) $x^2 + 3z^2 = 2y^2 + 1$
   (b) $x^2 + 3z^2 = 2y^2$
   (c) $x^2 + 3z^2 = 2y^2 - 1$
   (d) $x^2 + 3z^2 + 2y^2 = 1$
   (e) $x^2 + 3z^2 + 2y^2 = 0$
   (f) $x^2 + 3z^2 + 2y^2 + 1 = 0$
   (g) $x - 2y^2 = 3z^2 + 1$
   (h) $x + 2y^2 = 3z^2 + 1$
   (i) $x + 2y^2 = 1$
   (j) $x + 2y + 3z^2 = 1$

2. A particle's velocity is given by $\mathbf{v}(t) = \langle 2t-1, 4t, 3\rangle$. At $t = 0$ the particle is observed at the point $(1, 2, 3)$. Write a vector-valued function giving the position of the particle at each t.

3. How far does the particle in problem 2 travel *along its arc* between $t = 1$ and $t = 3$?

4. Find a parameterization for the curve formed by the intersection of the two surfaces
$$x^2 + y^2 = 36 \quad \text{and} \quad 2x - 3y + 4z = 0.$$

5. The function $\mathbf{r}(t) = \langle t^3, t^2 - 1, 3t^2 + 2\rangle$ traces a space curve. Find a parameterization for *the line tangent to the curve* at the point $\mathbf{r}(1)$.

6. For the curve given in problem #5, find the curvature at the point $(-1, 0, 5)$. (Simplify a reasonable amount.)

7. Let $\mathbf{p}(t) = \langle 2t^2 - 1,\ t - 2,\ t - t^2\rangle$. Find a_T and a_N, the tangential and normal components of the acceleration vector $\mathbf{p}''(t)$.

8. Two particles travel along the following curves, parameterized by time t.

$$\mathbf{r}_1(t) = \langle t,\ t^2 + 3t,\ t^3\rangle \quad \text{and} \quad \mathbf{r}_2(t) = \langle t^2 - 6,\ 2t^2,\ 3t^2\rangle$$

   Find

   (a) all collisions and

   (b) all intersections

   of these two curves (if any). Give the (x, y, z) coordinates of the solutions. Show all relevant steps that must be used to solve problems of this type, even if you can use shortcuts or find the answers by inspection in this particular problem. You're trying to convince me that you can deal with more complicated examples.

9. Suppose a particle moves along a path whose space curve is differentiable and that the particle is constrained to move on the surface of a sphere centered at the origin. Prove that the particle's tangent vector is perpendicular to its position vector.

10. Choose either $f(x) = \sin x$ or $f(x) = x^3$ for this problem. Then let $\kappa(x)$ denote the curvature of the curve $y = f(x)$ at each x.

    (a) Find a formula for $\kappa(x)$.

    (b) Find the maximum value of $\kappa(x)$. (Prove your result, else there will be only small partial credit.)

$\star\star\star\star$ EXTRAS $\star\star\star\star$

(A) ($\star$) Show that the curve in problem #7 lies entirely in a plane. Find an equation for the plane in standard form.

(B) ($\star$) In fact, it turns out that the curve in problem #7 is a parabola. Find the vertex of that parabola.

(C) ($\star$) Give a *precise, mathematical* definition of an *evolute* of a plane curve. (Partial credit is possible, but full credit only comes with a full definition.)

(D) ($\star$) Prove that if $\mathbf{r}(t)$ is differentiable, then

$$\frac{d}{dt}\|\mathbf{r}(t)\| = \frac{\mathbf{r}(t)}{\|\mathbf{r}(t)\|} \cdot \mathbf{r}'(t)$$

wherever $\mathbf{r}(t) \neq 0$.

(E) ($\star\cdots\star$) Ask a question you wish I had asked and answer it. Points vary depending on the difficulty of the question (and the correctness of the solution). Very few points (if any) will be awarded for a problem that is essentially represented elsewhere on the test. (In other words, no repeats.)