Sorting between lines

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Let N be a positive integer and define the sequence $T_N = \{t_n : n = 1, 2, ..., N\}$, where each t_n is chosen at random (uniformly distributed) in the interval [n, 2n]. Let $S_N = \{s_n : n = 1, 2, ..., N\}$ denote the sequence of sorted values of T_N (in increasing order). For large N, when the points $\{(n, s_n) : n = 1, 2, ..., N\}$ are plotted in the plane, a smooth curve seems to emerge. Prove that this is indeed the case by showing that the set of normalized points

$$\left\{ \left(\frac{n}{N}, \frac{s_n}{N}\right) : n = 1, 2, \dots, N \right\}$$

converges to points of the graph y = g(x) of a smooth function $g: [0,1] \rightarrow [0,2]$.

