11001. (*as actually proposed by Rick Mabry, LSU-Shreveport.*) A common problem in a calculus course is: If the real series $\sum_{n=1}^{\infty} a_n$ converges, does $\sum_{n=1}^{\infty} a_n^2$ necessarily converge? The answer is no, with the usual counterexample given by $a_n = (-1)^n/\sqrt{n}$. Instead, we propose the following variations.

a) Suppose that the real series $\sum_{n=1}^{\infty} a_n$ converges and let $p > 1$ be an odd integer. Show that the series

$$\sum_{n=1}^{\infty} a_n^p$$

does not necessarily converge.

b) Assume that the real series $\sum_{n=1}^{\infty} a_n$ converges. Does there necessarily exist a positive integer P such that

$$\sum_{n=1}^{\infty} a_n^p$$

converges for all odd $p > P$?

c) Assume that the alternating series $\sum_{n=1}^{\infty} (-1)^n a_n$ converges. Does there necessarily exist a positive integer P such that

$$\sum_{n=1}^{\infty} (-1)^n a_n^p$$

converges for all odd $p > P$?