Properties and Applications of Copulas: A Brief Survey

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Copulas are functions that join or "couple" multivariate distribution functions to their one-dimensional margins. Their importance in statistical modeling is primarily a consequence of *Sklar's Theorem*: Let H be a two-dimensional distribution function with marginal distribution functions F and G. Then there exists a copula C such that $H(x,y) = C(F(x),G(y))$. Conversely, for any distribution functions F and G and any copula C, the function H defined above is a two-dimensional distribution function with margins F and G. Furthermore, if F and G are continuous, C is unique.

In statistical modeling, dependence is often of more interest than independence, and many descriptions and measures of dependence are distribution free or scale invariant, and such properties and measures are expressible in terms of copulas.

In this talk, we explore the relationships among dependence concepts such as concordance, quadrant dependence, and likelihood ratio dependence, and measures of association such as the population versions of Spearman's rho, Kendall's tau, and Gini's gamma. The problem of finding best-possible bounds on certain sets of copulas leads to *quasi-copulas*, and we consider briefly some of their properties and applications.