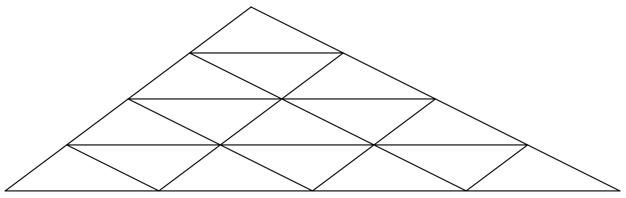
Congruent cakes (see Puzzle Corner 15, *Gaz. Aust. Math. Soc.* 36 (Nov. 2009), p. 311)

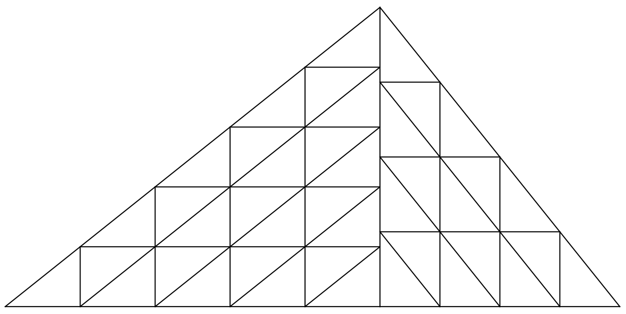
Show that it is possible to bake a triangular cake
and cut it into 2009 congruent triangular pieces.

Solution. (Don't stare at the large figures for too long - you'll get a headache!)

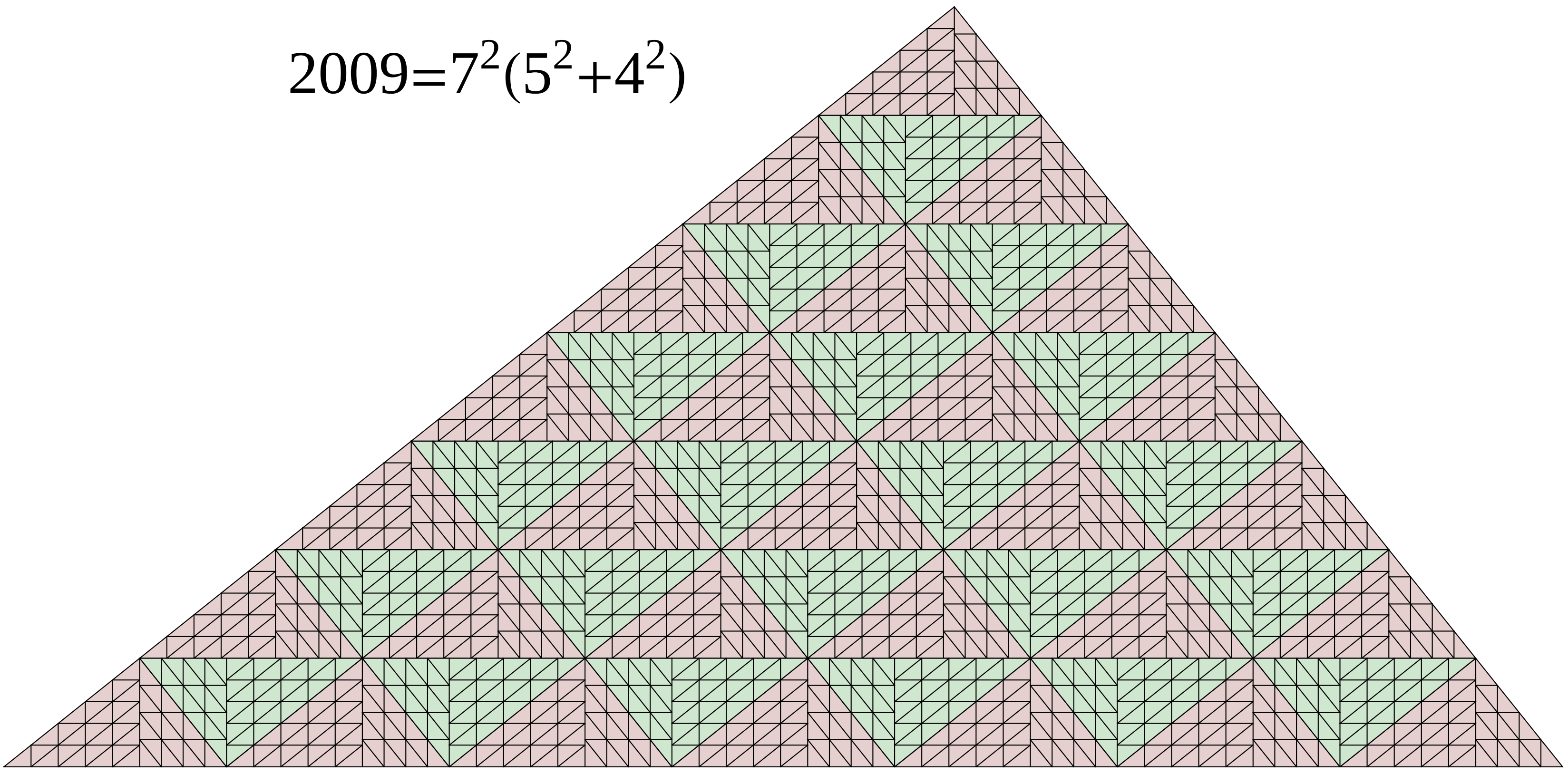
(1) Subdivide the edges of any triangle into k equal parts and connect the points
(as shown below with $k = 4$) to form k^2 congruent triangles similar to the original.
(Proofs of obvious geometric facts using similar triangles are omitted.)



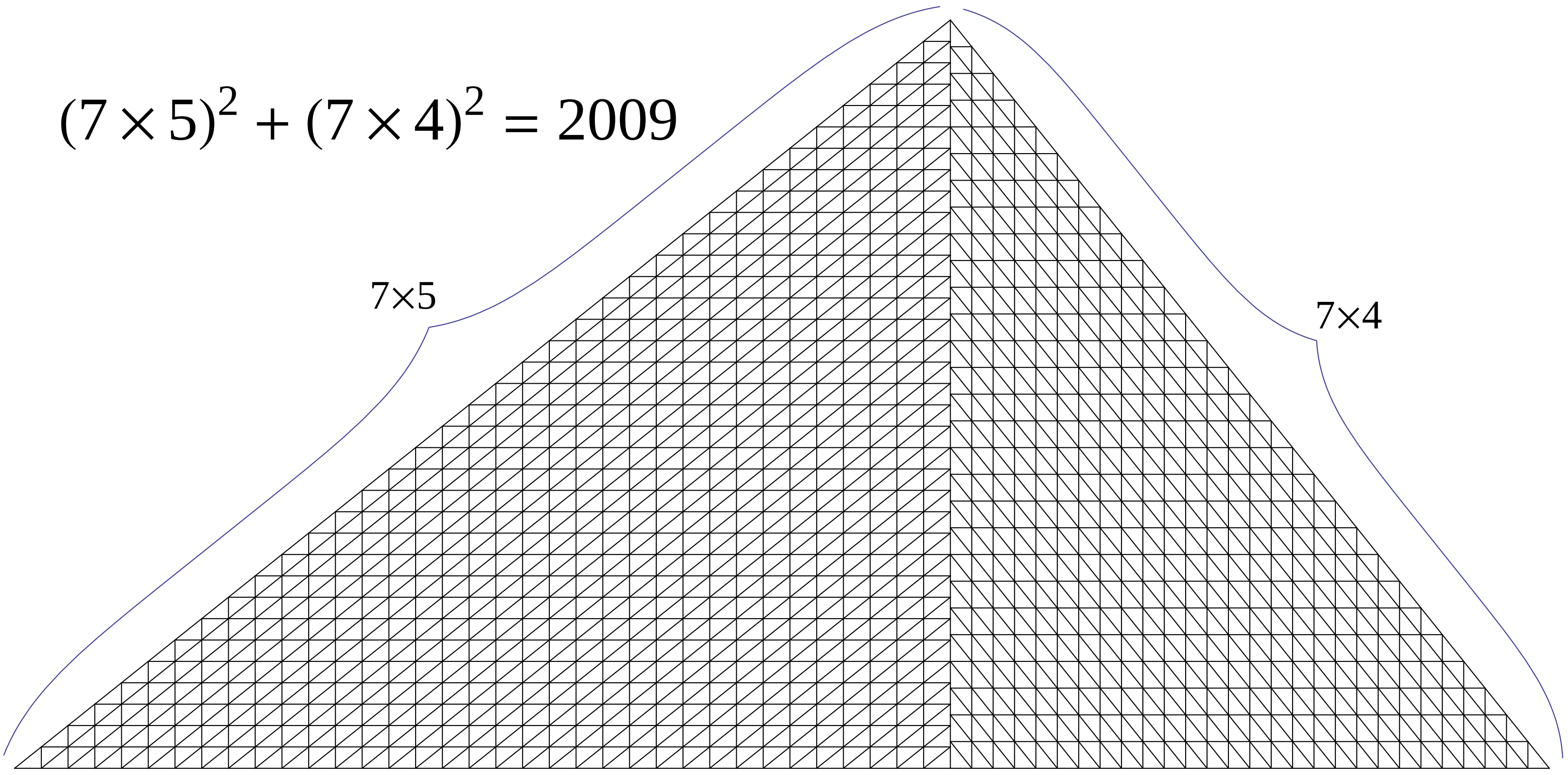
(2) Apply (1) to a pair of right triangles with legs of integral length m and n to form
$m^2 + n^2$ congruent triangles. Below, $m = 5$ and $n = 4$ give $5^2 + 4^2 = 41$ congruent triangles.



(3) Apply (1) to the result of (2) to make 7^2 copies of the above for $7^2\left(5^2 + 4^2\right) = 2009$
congruent triangles. (The following image *is* the solution.)



Of course, one could use (2) directly: $(7\times5)^2 + (7\times4)^2 = 2009$. But the figure above makes it
easy to count the triangles.



Rick Mabry, 16 Dec 2009. (Converted by *Mathematica* to PDF.)