PROBLEMS

10990. Proposed by Rick Mabry, LSUS, Shreveport, LA. The $n + 1$ Bernstein polynomials of degree $n$ are defined by

$$b_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}, \quad 0 \leq k \leq n.$$ 

When all $n + 1$ polynomials are plotted on the same graph for large fixed $n$ over the interval $0 \leq x \leq 1$, an ‘upper envelope’ begins to be seen. The figure below shows a scaled plot of the case $n = 16$, with the vertical scale multiplied by 4.

Let $\beta(x) = \lim_{n \to \infty} \sqrt{n} \max_{0 \leq k \leq n} b_{n,k}(x)$. Find a closed form expression for $\beta(x)$. 