Bernstein envelopes

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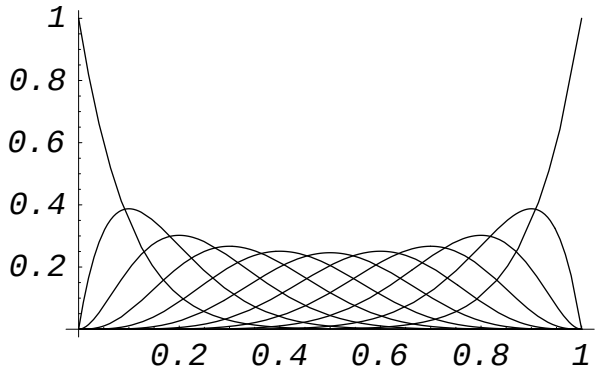
The $n+1$ Bernstein polynomials of degree n are defined by

$$b_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}, \quad k = 0, 1, 2, \ldots, n.$$

When all $n+1$ polynomials are plotted on the same graph for a large fixed n and $0 \le x \le 1$, an "upper envelope" begins to emerge.



$y = b_{n,k}(x), \quad 0 \le k \le n$ for $n = 10$.

In fact, for suitable constants B_n (depending only on n), the $n+1$ plots of

$$y = B_n b_{n,k}(x), \quad 0 \le x \le 1, \quad k = 0, 1, 2, \ldots, n$$

appear to approach a single (i.e., independent of n) fixed envelope $y = \beta(x)$. Find B_n and $\beta(x)$.

[Note to editor: The pictures herein are not necessary. They are included only as enticements.]