Q1003. *Proposed by Rick Mabry, Louisiana State University in Shreveport, Shreveport, LA.*

What are the zeros of the nth derivative of $f(x) = x^2 e^x$?

A1003.

It is clear that $f^{(n)}(x) = (a_n x^2 + b_n x + c_n)e^x$ for constants a_n, b_n, and c_n. Differentiating once gives the following simple recursive formulas: $(a_0, b_0, c_0) = (1, 0, 0)$, and $(a_{n+1}, b_{n+1}, c_{n+1}) = (a_n, 2a_n + b_n, c_n + b_n)$. Thus $a_n = 1$ for all n, $b_n = 2n$ for all n, and $c_j - c_{j-1} = 2(j-1)$ for all $j \geq 1$. Adding the last identity when $1 \leq j \leq n$ gives $c_n = n(n-1)$. Thus $f^{(n)}(x) = (x^2 + 2nx + n(n-1))e^x$, whose zeros are $-n \pm \sqrt{n}$.