PROBLEMS AND SOLUTIONS

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Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the inside front cover. Submitted solutions should arrive at that address before April 30, 2008. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11327. *Proposed by Rick Mabry and Debbie Shepherd, LSUS, Shreveport, LA.* A game of chance determines a sequence $\langle f_N \rangle$ of functions on [0, 1]. The Nth function f_N is constructed as follows: First, real numbers $t_N^1, \dots, t_N^N$ are chosen, independently, with t_N^n drawn at random from the interval $[n, 2n]$ with uniform distribution. The list $(t_N^1, \dots, t_N^N)$ is then sorted in increasing order to give a second list $(s_N^1, \dots, s_N^N)$. Finally, an increasing function f_N is defined on [0, 1] by connecting the dots (0, 0) and $(n/N, s_N^n/N)$ for $1 \le n \le N$. Show that there is a continuous function g from [0, 1] onto [0, 2] such that with probability 1, $\lim_{N\to\infty} \sup_{0\le x\le 1} |f_N(x) - g(x)| = 0$, and find a simple formula for g.

11328. *Proposed by Dmitris Vartziotis, Ioánnina, Greece.* Let $ABCD$ be a convex quadrilateral. Let P be the point outside $ABCD$ such that angle APB is a right angle and P is equidistant from A and B. Let points Q, R, and S be given by the same conditions with respect to the other three edges of $ABCD$. Let J, K, L, and M be the midpoints of PQ, QR, RS, and SP, respectively. Prove that $JKLM$ is a square.

11329. *Proposed by T. Amdeberhan and Victor H. Moll, Tulane University, New Orleans, LA.* Let $f(t) = 2^{-t} \ln \Gamma(t)$, where Γ denotes the classical *gamma function*, and let γ be Euler's constant. Derive the following integral identities:

$$\int_0^\infty f(t)\,dt = 2\int_0^1 f(t)\,dt - \frac{\gamma + \ln\ln 2}{\ln 2},$$

$$\int_0^\infty t f(t)\,dt = 2\int_0^1 (t+1) f(t)\,dt - \frac{(\gamma + \ln\ln 2)(1 + 2\ln 2) - 1}{\ln^2 2}\ .$$

11330. *Proposed by Marian Tetiva, National College "Gheorghe Roşca Codreanu", Bîrlad, Romania.* For a triangle with semiperimeter s, inradius r, circumradius R, and heights h_a, h_b, and h_c, show that

$$h_a + h_b + h_c - 9r \ge 2s\sqrt{\frac{2r}{R}} - 6\sqrt{3}\,r.$$