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-**Proof of Fact 5.** Holding θ and r constant, let $f(y) = x(y, \theta, r)$ and differentiate (A) 408 and (B) with respect to y to get

$$f'(y) = -\tan \alpha + (1-y)\sec^2 \alpha \frac{d\alpha}{dy}$$

and

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$$\frac{d\alpha}{dy} = \frac{\sin\theta}{2r}\sec(\theta + \alpha)$$

605 Differentiating each once more gives 100

$$f''(y) = -2\sec^2 \alpha \frac{d\alpha}{dy} + (1-y)\sec^2 \alpha \left(2\tan\alpha \left(\frac{d\alpha}{dy}\right)^2 + \frac{d^2\alpha}{dy^2}\right)$$

and 412

$$\frac{d^2\alpha}{dy^2} = \frac{\sin\theta}{2r}\sec(\alpha+\theta)\tan(\alpha+\theta)\frac{d\alpha}{dy}$$

Combining these gives 405

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$$\frac{d^2x}{dy^2} = \frac{d\alpha}{dy}(\sec^2\alpha)(-2+\mathcal{J}),$$

where

$$\mathcal{J} = (1 - y)\frac{\sin\theta}{2r}\sec(\alpha + \theta)(2\tan\alpha + \tan(\alpha + \theta)).$$

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We'll show that $\mathcal{J} < 3/2$, which will prove that f''(y) < 0. Since $\tan \alpha < \tan(\alpha + \theta)$, we have

$$\sec(\alpha + \theta)(2\tan\alpha + \tan(\alpha + \theta)) < 3\sec(\alpha + \theta)\tan(\alpha + \theta)$$
$$= \frac{3\sin(\alpha + \theta)}{1 - \sin^2(\alpha + \theta)}$$
$$= \frac{3(\sin\theta)(1 + \frac{y}{2r})}{1 - (\sin^2\theta)(1 + \frac{y}{2r})^2}.$$

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This implies that

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$$\begin{aligned} \mathcal{I} &< \frac{(1-y)}{2r} \frac{3\left(\frac{2r}{1+2r}\right)^2 \left(1+\frac{y}{2r}\right)}{1-\left(\frac{2r}{1+2r}\right)^2 \left(1+\frac{y}{2r}\right)^2} \\ &= \frac{3(2r+y)}{1+4r+y} \\ &< \frac{3}{2}. \end{aligned}$$

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