Proof of Fact 8. Since $f(1-y) > f(y) \Leftrightarrow \frac{f(1-y)}{y(1-y)} > \frac{f(y)}{y(1-y)}$, by defining $g(y) = \frac{f(y)}{y(1-y)}$, it suffices then to show that g is increasing on $[0, 1/2]$. Using $\sin(\theta+\alpha) = \left(1 + \frac{y}{2r}\right)\sin\theta$, or $\sin(\theta+\alpha) - \sin\theta = \frac{y}{2r}\sin\theta$, and $\sin\theta \leq \frac{2r}{y+2r}$, we have,

$$
\begin{aligned}
g'(y) &= \frac{f'(y)}{y(1-y)} + \frac{f(y)(2y-1)}{y^2(1-y)^2} \\
&= \frac{-y\tan\alpha + y(1-y)(\sec^2\alpha)\frac{\sin\theta}{r}\sec(\theta+\alpha) + (2y-1)\tan\alpha}{y^2(1-y)} \\
&= \frac{y\sec^2\alpha\frac{\sin\theta}{r}\sec(\theta+\alpha) - \tan\alpha}{y^2} \\
&= \frac{\sec^2\alpha(\sin(\theta+\alpha) - \sin\theta)\sec(\theta+\alpha) - \tan\alpha}{y^2} \\
&> \frac{(\sin(\theta+\alpha) - \sin\theta)\sec(\theta+\alpha) - \tan\alpha}{y^2} \\
&= \frac{\sec\alpha\sec(\theta+\alpha)}{y^2}((\sin(\theta+\alpha) - \sin\theta)\cos\alpha - \cos(\theta+\alpha)\sin\alpha) \\
&= \frac{\sec\alpha\sec(\theta+\alpha)}{y^2}\sin\theta(1-\cos\alpha) \\
&> 0.
\end{aligned}
$$